Answer on Question #65803 – Math – Calculus

Question

If $u = x\cos y,\ v = ye^x,\ w = \sin(xz)$, find the Jacobian $\partial(u,v,w)/\partial(x,y,z)$, and evaluate it at $\left(2, 0, \frac{\pi}{3}\right)$.

Solution

The Jacobian $\partial(u,v,w)/\partial(x,y,z)$ is the determinant

$$J = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}.$$

Note that

$$\frac{\partial u}{\partial x} = \cos y;\ \frac{\partial u}{\partial y} = -x\sin y;\ \frac{\partial u}{\partial z} = 0;$$

$$\frac{\partial v}{\partial x} = ye^x;\ \frac{\partial v}{\partial y} = e^x;\ \frac{\partial v}{\partial z} = 0;$$

$$\frac{\partial w}{\partial x} = z\cos(xz);\ \frac{\partial w}{\partial y} = 0;\ \frac{\partial w}{\partial z} = x\cos(xz);$$

Hence

$$J = \begin{vmatrix} \cos y & -x\sin y & 0 \\ ye^x & e^x & 0 \\ z\cos(xz) & 0 & x\cos(xz) \end{vmatrix} =$$

$$= x\cos(xz)\cdot \begin{vmatrix} \cos y & -x\sin y \\ ye^x & e^x \end{vmatrix} = x\cos(xz)(e^x\cos y + xye^x\sin y) =$$

$$= xe^x\cos(xz)(\cos y + xy\sin y).$$

To evaluate the Jacobian at the point $\left(2, 0, \frac{\pi}{3}\right)$ we substitute $x = 2,\ y = 0,\ z = \frac{\pi}{3}$:

$$J = 2\cdot e^2\cdot\cos\left(\frac{2\pi}{3}\right)(\cos 0 + 2\cdot 0\cdot\sin 0) = 2\cdot e^2\cdot\left(-\frac{1}{2}\right)(1+0) = -e^2.$$

Answer: $J = xe^x\cos(xz)(\cos y + xy\sin y);\ -e^2.$