# Answer on Question #65801 – Math – Calculus

## Question

Find the second Taylor polynomial of f(x,y)=1+3x^2y-4y^3 at (1,1).

## Solution

By the definition, the second Taylor polynomial of $f(x,y)$ at $(1,1)$ is given by

$$T_2(x,y) = f(1,1) + (x-1)\frac{\partial f(1,1)}{\partial x} + (y-1)\frac{\partial f(1,1)}{\partial y} + \frac{1}{2!}\left[(x-1)^2\frac{\partial^2 f(1,1)}{\partial x^2} + 2(x-1)(y-1)\frac{\partial^2 f(1,1)}{\partial x \partial y} + (y-1)^2\frac{\partial^2 f(1,1)}{\partial y^2}\right]$$

Here

$$f(x,y) = 1 + 3x^2y - 4y^3, \qquad f(1,1) = 1 + 3 - 4 = 0;$$

$$\frac{\partial f(x,y)}{\partial x} = 6xy, \qquad \frac{\partial f(1,1)}{\partial x} = 6;$$

$$\frac{\partial f(x,y)}{\partial y} = 3x^2 - 12y^2, \qquad \frac{\partial f(1,1)}{\partial y} = 3 - 12 = -9;$$

$$\frac{\partial^2 f(x,y)}{\partial x^2} = 6y, \qquad \frac{\partial^2 f(1,1)}{\partial x^2} = 6;$$

$$\frac{\partial^2 f(x,y)}{\partial x \partial y} = 6x, \qquad \frac{\partial^2 f(1,1)}{\partial x \partial y} = 6;$$

$$\frac{\partial^2 f(x,y)}{\partial y^2} = -24y, \qquad \frac{\partial^2 f(1,1)}{\partial y^2} = -24.$$

So, $T_2(x,y) = 6(x-1) - 9(y-1) + \frac{1}{2!}[6(x-1)^2 + 2\cdot 6(x-1)(y-1) - 24(y-1)^2] =$ $6(x-1) - 9(y-1) + 3(x-1)^2 + 6(x-1)(y-1) - 12(y-1)^2$.

**Answer:**

The second Taylor polynomial of $f(x,y) = 1 + 3x^2y - 4y^3$ at $(1,1)$ is

$$6(x-1) - 9(y-1) + 3(x-1)^2 + 6(x-1)(y-1) - 12(y-1)^2.$$