

## Answer on Question #65798 – Math – Calculus

### Question

If  $f(x, y) = x^{\frac{1}{4}} + \frac{y^{\frac{1}{4}}}{x^{\frac{1}{5}}} + y^{\frac{1}{5}}$  then show that  $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = \frac{1}{20} f(x, y)$

### Solution

If  $f(x, y) = x^{\frac{1}{4}} + \frac{y^{\frac{1}{4}}}{x^{\frac{1}{5}}} + y^{\frac{1}{5}}$ , then

$$\frac{\partial f}{\partial x} = \frac{1}{4}x^{-\frac{3}{4}} - \frac{1}{5}y^{\frac{1}{4}}x^{-\frac{6}{5}}, \quad \frac{\partial f}{\partial y} = \frac{1}{4}y^{-\frac{3}{4}}x^{-\frac{1}{5}} + \frac{1}{5}y^{-\frac{4}{5}}.$$

The first summand of the sum is

$$x \frac{\partial f}{\partial x} = x \left( \frac{1}{4}x^{-\frac{3}{4}} - \frac{1}{5} \frac{y^{\frac{1}{4}}}{x^{\frac{6}{5}}} \right) = \frac{1}{4}x^{\frac{1}{4}} - \frac{1}{5} \frac{y^{\frac{1}{4}}}{x^{\frac{1}{5}}}$$

The second summand of the sum is

$$y \frac{\partial f}{\partial y} = y \left( \frac{1}{5}y^{-\frac{4}{5}} + \frac{1}{4} \frac{y^{-\frac{3}{4}}}{x^{\frac{1}{5}}} \right) = \frac{1}{5}y^{\frac{1}{5}} + \frac{1}{4} \frac{y^{\frac{1}{4}}}{x^{\frac{1}{5}}}$$

The sum is given by

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = \frac{1}{4}x^{\frac{1}{4}} - \frac{1}{5} \frac{y^{\frac{1}{4}}}{x^{\frac{1}{5}}} + \frac{1}{5}y^{\frac{1}{5}} + \frac{1}{4} \frac{y^{\frac{1}{4}}}{x^{\frac{1}{5}}} = \frac{1}{4}x^{\frac{1}{4}} + \frac{1}{20} \frac{y^{\frac{1}{4}}}{x^{\frac{1}{5}}} + \frac{1}{5}y^{\frac{1}{5}} = \left( 5x^{\frac{1}{4}} + \frac{y^{\frac{1}{4}}}{x^{\frac{1}{5}}} + 4y^{\frac{1}{5}} \right) \frac{1}{20} \neq \frac{1}{20} f(x, y)$$

It is easy to see that the statement of question is not correct.