Answer on Question #65751 – Math – Other

Question

Solve the following linear programming problem graphically:

Maximize $f(x_1, x_2) = x_1 + 3x_2$

Subject to:

$-3x_1 + 7x_2 \leq 21$

$3x_1 - 3x_2 \leq 3$

$-2x_1 + 2x_2 \leq 4$

$x_{1,} x_2 \geq 0$

Solution

Let us represent the feasible region.

Graph [1]: $-3x_1 + 7x_2 \leq 21$

Graph the line $-3x_1 + 7x_2 = 21$

It has intercepts (-7,0) and (0,3). Mark them on the graph and draw a straight line across them.

Shade the region below the line.

Graph [2]: $3x_1 - 3x_2 \leq 3$

The simplified form of $3x_1 - 3x_2 = 3$ is $x_1 - x_2 = 1$. Graph the line.

It has intercepts (1,0) and (0,-1). Mark them on the graph and draw a straight line across them. Shade the region above the line.

Graph [3]: $-2x_1 + 2x_2 \leq 4$

The simplified form of $-2x_1 + 2x_2 = 4$ is $-x_1 + x_2 = 2$. Graph the line.

It has intercepts (-2,0) and (0,2). Mark them on the graph and draw a straight line across them.

Shade the region below the line.

Graph [4]: $x_1 \geq 0$

Shade the region to the right of the x_1-axis.

Graph [5]: $x_2 \geq 0$

Shade the region above the x_2-axis.

The feasible region is the intersection of the regions defined by the set of constraints and the coordinate axis (conditions of non-negativity of variables). This feasible region is represented at the picture by the polygon OABCD in blue color.

We want to know vertices of the shaded region.
We can see three of them: O(0,0), D(1,0), A(0,2).

The fourth is the intersection of the first two lines. Let us solve the system:

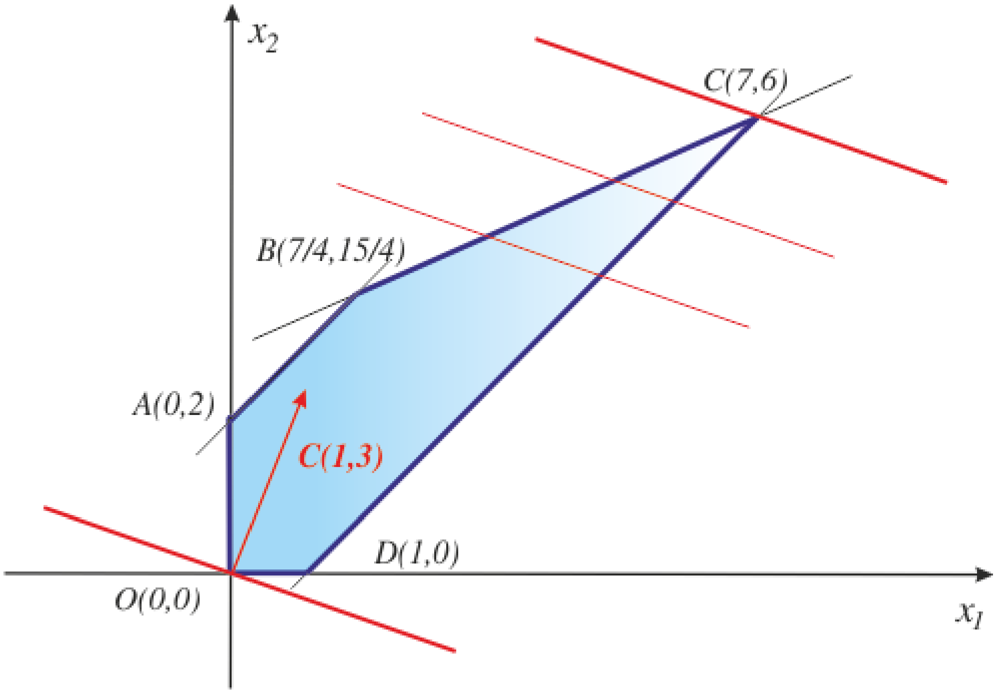
$$\begin{cases} -3x_1 + 7x_2 = 21 \\ 3x_1 - 3x_2 = 3 \end{cases}$$

and we get C (7,6).

The fifth vertex is the intersection of the first and the third lines. Let us solve the system:

$$\begin{cases} -3x_1 + 7x_2 = 21 \\ -x_1 + x_2 = 2 \end{cases}$$

and we get B $\left(\frac{7}{4}, \frac{15}{4}\right)$.



Now let us draw objective function line.

Objective function line of $x_1 + 3x_2$=0 goes through the origin and is coloured in red.

Optimum point of a linear programming problem always lies on one of the corner points (vertices) of the graph's feasible region.

To find the optimum point, we need to slide a ruler across the graph along the gradient of objective function. In our case the gradient $\left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}\right) = C(1,3)$. Where the objective is to maximize, we must slide the ruler up to the point within the feasible region which is furthest away from the origin. In our case the vertex C(7,6) is the optimum point.

The maximum of the function f in the feasible region is $7 + 3 \cdot 6 = 25$.

A graphical method of linear programming can be found at

http://accounting-simplified.com/management/limiting-factor-analysis/linear-programming/graphical.html

Answer: $max\ f = f(7,6) = 25$.