Question #65710, Math / Statistics and Probability

A lawyer commutes daily from his suburban home to his midtown office. On the average the trip one way takes 24 minutes, with a standard deviation of 3.8 minutes. Assume the distribution of trip times to be normally distributed.

a) what is probability that a trip will take at least 1/2 hour?

Solution

$$z = \frac{x-\mu}{\sigma};$$

$$z = \frac{30-24}{3.8} = 1.58$$

$p(z > z_0) = 1 - p(z < z_0)$

The cumulative probability value associated with the given *z*-score can be either obtained from the standard normal table or calculated using the technology (NORM.S.DIST() function of MS Excel).

$p(z < 1.58) = 0.9428;$

$p(z > 1.58) = 1 - 0.9428 = 0.0572$

b) If the office opens at 9:00 a.m. and he leaves his house at 8:45 a.m. daily what percentage of the time is he late for work?

Solution

To answer the question, one needs to determine the probability that the commute time will exceed 15 minutes.

$$z = \frac{15-24}{3.8} = -2.37$$

$p(z < -2.37) = 0.0089$

$p(z > -2.37) = 1 - 0.0089 = 0.9911$

The lawyer will be late 99.11% of the time.

c) if he leaves the house at 8:35 a.m and coffee is served at the office from 8:50 a.m until 9:00 a.m. , what is the probability that he misses coffee?

Solution

Since there is no restriction on the time required to drink coffee, and it is served till 9:00, to answer the question, one needs to determine the probability that the commute time will exceed 25 minutes.

$$z = \frac{25-24}{3.8} = 0.26$$

$p(z < 0.26) = 0.6038$

$p(z > 0.26) = 1 - 0.\ 6038 = 0.3962$

The probability of missing the coffee is 0.3962.

d) find the length of time above which we find the slowest 15% of the trips.

Solution

The given length corresponds to the fastest 85% of the trips.

The *z*-score associated with the given cumulative probability value can be either obtained from the standard normal table or calculated using the technology (NORM.S.INV() function of MS Excel).

For $p = 0.85$, $z = 1.04$.

Converting the *z*-score to the data score:

$x = \mu + z\sigma$;

$x = 24 + 1.04 \times 3.8 = 27.94$

The slowest 15% of the trips will take more than 28 minutes.

e) find the probability that 2 of the next 3 trips will take at least 1/2 hour?

Solution

The number of trips that will take at least 30 minutes represents the binomial random variable with $p = 0.0572$ and $n = 3$.

Using the binomial formula:

$$P(x) = \frac{n!}{(n-x)!x!} p^x q^{n-x};$$

$$P(2) = \frac{3!}{(3-2)!2!} 0.0572^2 \times (1-0.0572)^{3-2} = 0.0093$$