Answer on Question #65690 – Math – Linear Algebra

Question

Let V be a vector space over a field F and let $T : V \to V$ be a linear operator.
Show $T(W) \subset W$ for any subspace W of V if and only if there is a $\lambda \in F$ such that $Tv = \lambda v$ for all $v \in V$.

Solution

Necessity. Assume that $T : V \to V$ be a linear operator and $T(W) \subset W$ for any subspace W of V. Consider any vector $v \in V$. By assumption, the subspace image

$T(\langle v \rangle) \subset \langle v \rangle$ is a subset of the subspace.

Thus, the definition of $\langle v \rangle$ implies that

$$Tv = \lambda v$$

for some $\lambda \in F$.

Sufficiency. Assume that $T : V \to V$ be a linear operator and for all $v \in V$ there exists $\lambda \in F$:

$$Tv = \lambda v.$$

Suppose further W is subspace of V and $w \in W$.
From the definition of a linear space, the assumption implies that there exists $\lambda \in F$:

$$Tw = \lambda w \in W.$$

So $T(W) \subset W$.