Answer on Question #65658, Math / Calculus

A price p (in dollars) and demand x for a product are related by:

$$2x^2 + 5xp + 50p^2 = 24800\,.$$

If the price is increasing at a rate of 2 dollars per month when the price is 20 dollars, find the rate of change of the demand.

Solution

A price p and demand x are functions of time t, dp/dt and dx/dt their rates of change over time. We have that

$$2x^2 + 5xp + 50p^2 = 24800, x > 0$$

Differentiate on both sides of the equation with respect to t

$$\frac{d}{dt}(2x^2 + 5xp + 50p^2) = \frac{d}{dx}(24800)$$

Use Chain rule and Product

$$2(2x)\frac{dx}{dt} + 5p\frac{dx}{dt} + 5x\frac{dp}{dt} + 50(2p)\frac{dp}{dt} = 0$$

Solve for dx/dt

$$\frac{dx}{dt} = -5\frac{x + 20p}{4x + 5p} \cdot \frac{dp}{dt}$$

Find demand x for a product when the price is 20 dollars

$2x^2 + 5(20)x + 50(20)^2 = 24800, x > 0$

$2x^2 + 100x - 4800 = 0, x > 0$

$x^2 + 50x - 2400 = 0, x > 0$

$(x + 80)(x - 30) = 0, x > 0$

$x = 30\ dollars$

To find the of change of the demand let

$$x = \$30, p = \$20, \text{and } dp/dt = 2\text{dollars per month}$$

Then

$$\frac{dx}{dt} = -5\frac{30 + 20(20)}{4(30) + 5(20)} \cdot 2 = -\frac{215}{11}\left(\frac{dollars}{month}\right) \approx 19.55\frac{dollars}{month}$$

The demand is decreasing at a rate of approximately 19.55 dollars per month.