Answer on Question #65647 – Math – Calculus

Question

Calculate
lim x tends ∞− x^2 ln(x^2+1/x^2)^3

Solution

Method 1

It is known that

$$\lim_{y\to\infty}\left(1+\frac{1}{y}\right)^{y}=e,$$

hence

$$\lim_{y\to\infty}\left(1+\frac{1}{y}\right)^{y}=\left|z=\frac{1}{y}\to 0\ as\ y\to\infty\right|=\lim_{z\to 0}(1+z)^{\frac{1}{z}}=e\ \text{ and}$$

using the previous formula, the continuity of the logarithmic function one gets

$$1=ln\,(e)=ln\,\left(\lim_{z\to 0}(1+z)^{\frac{1}{z}}\right)=\lim_{z\to 0}ln\,(1+z)^{\frac{1}{z}}=\lim_{z\to 0}\frac{1}{z}ln(1+z).$$

Thus,

$$\lim_{z\to 0}\frac{ln(1+z)}{z}=1 \qquad (1)$$

$$\lim_{x\to\infty}x^2ln\left(\frac{x^2+1}{x^2}\right)^3=3\lim_{x\to\infty}x^2ln\left(\frac{x^2+1}{x^2}\right)=3\lim_{x\to\infty}\frac{ln\left(1+\frac{1}{x^2}\right)}{\frac{1}{x^2}}=\left|t=\frac{1}{x^2}\to 0\ as\ x\to\infty\right|$$

$$=3\lim_{t\to 0}\frac{ln\,(1+t)}{t}=|formula\ (1)|=3\cdot 1=3$$

Method 2

$$\lim_{x\to\infty}x^2ln\left(\frac{x^2+1}{x^2}\right)^3=3\lim_{x\to\infty}x^2ln\left(\frac{x^2+1}{x^2}\right)=3\lim_{x\to\infty}\frac{ln\left(1+\frac{1}{x^2}\right)}{\frac{1}{x^2}}=|L'H\hat{o}pital's\ rule|$$

$$=3\lim_{x\to\infty}\frac{\frac{1}{1+\frac{1}{x^2}}\left(-\frac{2}{x^3}\right)}{-\frac{2}{x^3}}=3\lim_{x\to\infty}\frac{1}{1+\frac{1}{x^2}}=\frac{3}{1+0}=3.$$

Answer: 3.

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