Question #65545, Math / Other

In queueing theory, if the arrivals are according to a Poisson distribution with parameter λ , the inter-arrival time is according to an exponential distribution with parameter eλ. State whether the statement is true or false. Justify your answer briefly.

Answer.
Statement is true.
The Poisson provides an appropriate description of the number of occurrences per interval of time, and then the exponential will provide a description of the length of time between occurrences. In a Poisson process, if events occur on average at the rate of λ per unit of time, then there will be on average λt occurrences per t units of time. The Poisson distribution describing this process is therefore $P(x) = e^{-\lambda t}\frac{(\lambda t)^k}{k!}$ **, from which** $P(x=0) = e^{-\lambda t}$**, is the probability of no occurrences in t units of time.**
Another interpretation of $P(x=0) = e^{-\lambda t}$ **is that this is the probability that the time, T, to the first occurrence is greater than t, i.e.**
$P(t > T) = P(x=0) = e^{-\lambda t}$.
Conversely, the probability that an event does occur during t units of time is given by $P(t \le T) = 1 - P(x=0) = 1 - e^{-\lambda t}$**.**
This is the cumulative exponential distribution which, when differentiated with respect to t, produces the probability density function of the exponential distribution $f(t) = \lambda e^{-\lambda t}$**.**