Answer on Question#65391 – Math – Statistics and Probability

Question. If the second moment of a Poisson distribution is 6, find the probability $P(X \geq 2)$.

Solution. Let X be the random variable with rate λ. It is well known that

$$E(X) = Var(X) = \lambda > 0$$

(see https://en.wikipedia.org/wiki/Poisson_distribution). By the condition we have:

$\sum_{k=0}^{\infty} k^2 P(X = k) = \sum_{k=0}^{\infty} k^2 \cdot \frac{\lambda^k}{k!} e^{-\lambda} = 6$

(for example see http://www.umsl.edu/~fraundorfp/ifzx/moments.html).

Since $Var(X) = E(X^2) - [E(X)]^2 = \sum_{k=0}^{\infty} k^2 \cdot \frac{\lambda^k}{k!} e^{-\lambda} - \lambda^2$

(see https://en.wikipedia.org/wiki/Variance) then we have the following equation:

$\lambda = 6 - \lambda^2 \Leftrightarrow \lambda^2 + \lambda - 6 = 0$. Solving it

(see http://www.wikihow.com/Solve-Quadratic-Equations) we obtain: $\left[\begin{array}{l} \lambda_1 = -3 < 0 \\ \lambda_2 = 2 \end{array}\right.$.

Since $\lambda > 0$ then we conclude that $\lambda = 2$, and X has the following distribution:

$$P(X = k) = \frac{2^k}{k!} e^{-2}, k = 0, 1, 2, \ldots$$

Then $P(X \geq 2) = 1 - P(X < 2) = 1 - [P(X = 0) + P(X = 1)] = 1 - e^{-2} - 2e^{-2} =$

$= 1 - 3e^{-2} = 1 - \frac{3}{e^2} = \frac{e^2 - 3}{e^2} \approx 0.594 = 59.4\%$.

Answer. $\frac{e^2-3}{e^2}$.