

Answer on Question #65109 – Math – Statistics and Probability

Question

The distribution of IQ for 12-years-old is known to be normal with mean 100 and standard deviation 16.

What must then be the probability for a 12-years-old to have an IQ of 124 or more?

What is the probability for a 12-years-old to have an IQ of 84 or less?

Solution

Let ξ be the random variable which means an IQ for 12-years-old person. It is known that ξ has a normal distribution [1] with mean $\mu = 100$ and standard deviation $\sigma = 16$, that is

$$\xi \sim N(100, 256),$$

where $\mu = 100$, $\sigma^2 = 256$.

Then the following random variable

$$\eta = \frac{\xi - \mu}{\sigma} = \frac{\xi - 100}{16} \sim N(0, 1)$$

has the standard normal distribution with mean 0 and standard deviation 1.

Thus the probability for a 12-years-old to have an IQ of 124 or more is equal to

$$P(\xi \geq 124) = P\left(\frac{\xi - 100}{16} \geq \frac{124 - 100}{16}\right) = P(\eta \geq 1.5).$$

Using the Standard Normal Distribution Table [2] we obtain:

$$P(\eta \geq 1.5) = 0.5 - P(0 \leq \eta \leq 1.5) = 0.5 - 0.4332 = 0.0668 = 6.68\%.$$

Similarly the probability for a 12-years-old to have an IQ of 84 or less is equal to

$$\begin{aligned} P(\xi \leq 84) &= P\left(\frac{\xi - 100}{16} \leq \frac{84 - 100}{16}\right) = P(\eta \leq -1) = P(\eta \geq 1) = 0.5 - P(0 \leq \eta \leq 1) = \\ &= 0.5 - 0.3413 = 0.1587 = 15.87\%. \end{aligned}$$

Answer: 6.68%; 15.87%.

References:

1. WolframMathWorld. Normal Distribution. Retrieved from <http://mathworld.wolfram.com/NormalDistribution.html>.
2. Standard Normal Distribution Table. Retrieved from <https://www.mathsisfun.com/data/standard-normal-distribution-table.html>

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