Answer on Question #65070 – Math – Statistics and Probability

Question

Let A and B be two events associated with an experiment such that

$$\mathrm{P}(\mathrm{A}) = 0.4$$

and

$$\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = 0.7.$$

Compute $\mathrm{P}(\mathrm{B})$ when

i) A and B are mutually exclusive;

ii) A and B are independent.

Solution

i) Recall that, for any two events A and B, we have *the inclusion-exclusion formula* [1, p.29] for probabilities:

$$\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B}) - \mathrm{P}(\mathrm{A} \cap \mathrm{B}).$$

Assume that events A and B are *mutually exclusive* [1, p.24].
Then by definition,

$$\mathrm{P}(\mathrm{A} \cap \mathrm{B}) = 0.$$

Hence the following formula can be obtained:

$$\mathrm{P}(\mathrm{B}) = \mathrm{P}(\mathrm{A} \cup \mathrm{B}) - \mathrm{P}(\mathrm{A}).$$

So,

$$\mathrm{P}(\mathrm{B}) = 0.7 - 0.4 = 0.3.$$

ii) Again, we can apply *the inclusion-exclusion formula* [1, p.29]:

$$\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B}) - \mathrm{P}(\mathrm{A} \cap \mathrm{B}).$$

By the definition of independence, two events A and B are *independent* [1, p.79] if and only if

$$\mathrm{P}(\mathrm{A} \cap \mathrm{B}) = \mathrm{P}(\mathrm{A})\mathrm{P}(\mathrm{B}).$$

Thus,

$$\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B}) - \mathrm{P}(\mathrm{A})\mathrm{P}(\mathrm{B}).$$

Now, express this formula in terms of $\mathrm{P}(\mathrm{B})$, bearing in mind that $\mathrm{P}(\mathrm{A}) \neq 1$:

$$\mathrm{P(B)} = \frac{\mathrm{P(A \cup B)} - \mathrm{P(A)}}{1 - \mathrm{P(A)}}.$$

Finally, we can compute the answer:

$$\mathrm{P(B)} = (0.7 - 0.4)/(1 - 0.4) = 0.3/0.6 = 0.5.$$

References:

1. Sheldon M. Ross. *A First Course in Probability.* 8th edition.

2. Richard N. Aufmann, Joanne S. Lockwood, Richard D. Nation, Daniel K. Clegg. *Mathematical Excursions*. 2nd edition.

3. Charles Henry Brase, Corrinne Pellillo Brase. *Understandable Statistics: Concepts and Methods*. 10th edition.

Answer:

i) $\mathrm{P(B)} = 0.3.$

ii) $\mathrm{P(B)} = 0.5.$