

Answer on Question #64948 – Math – Statistics and Probability

Question

3 batteries are chosen at random from 15 batteries from which 5 are defective. Find the probability that:

- A) none of the three are defective;
- B) exactly one is defective;
- C) 2 defective and one nondefective;
- D) at least one is nondefective.

Solution

A) Let's take the first battery from 15 batteries and denote the event by A. The probability of chosen battery not being defective will be $10/15$.

Let's take the next battery from other 14 batteries and denote the event by B. The probability of chosen battery not being defective will be $9/14$.

Let's take the last battery from remaining 13 batteries and denote the event by C. The probability of that battery not being defective will be $8/13$.

The probability of none of the three being defective is calculated as the product

$$P_1 = P(3 \text{ nondefective}) = P(A, B, C) = \frac{10}{15} \cdot \frac{9}{14} \cdot \frac{8}{13} = \frac{720}{2730} \approx 0.264$$

B) The number of combinations of choosing 3 batteries from 15 is C_{15}^3
 C_5^1 is the number of combinations of choosing 1 defective battery from 5 defective ones.
 C_{10}^2 is the number of combinations of choosing 2 good batteries from 10 good ones.
So $C_5^1 C_{10}^2$ is the total amount of combinations to choose exactly one defective and two good batteries. The probability of exactly one battery being defective will be

$$P_2 = P(2 \text{ nondefective}, 1 \text{ defective}) = \frac{C_5^1 C_{10}^2}{C_{15}^3} = \frac{5! \cdot 10!}{1! 4! 2! 8!} = \frac{5 \cdot 10 \cdot 9}{15 \cdot 14 \cdot 13}$$
$$= \frac{3 \cdot 5 \cdot 9 \cdot 10}{13 \cdot 14 \cdot 15} = \frac{45}{91}$$

C) The arguments are similar to ones in the previous part B) and the probability of choosing 2 from 5 defective batteries and 1 from 10 good batteries will be

$$P_3 = P(1 \text{ nondefective}, 2 \text{ defective}) = \frac{C_5^2 C_{10}^1}{C_{15}^3} = \frac{5! \cdot 10!}{2! 3! 1! 9!} = \frac{5 \cdot 4}{2} \cdot 10$$

$$= \frac{10 \cdot 3 \cdot 4 \cdot 5}{13 \cdot 14 \cdot 15} = \frac{20}{91}$$

D)

$$P_4 = P(\text{at least 1 is nondefective}) = P(1 \text{ nondefective}, 2 \text{ defective}) +$$

$$+ P(2 \text{ nondefective}, 1 \text{ defective}) + P(3 \text{ nondefective}) = P_3 + P_2 + P_1 =$$

$$= \frac{20}{91} + \frac{45}{91} + \frac{720}{2730} \approx 0.978.$$

Answer:

- A) 0.264 ;
- B) $\frac{45}{91}$;
- C) $\frac{20}{91}$;
- D) 0.978.