Answer on Question #64896 – Math – Statistics and Probability

Question

A and B are equally good tennis players. Which of the following two events is more probable?

1) A beats B exactly in 3 games out of 4.

2) A beats B exactly in 5 games out of 8.

Solution

We need to use binomial distributions with $p = \frac{1}{2}, n = 4$ and $p = \frac{1}{2}, n = 8$.

1)

$$P(X = 3) = \frac{4!}{3!\,(4-3)!}\left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{4-3} = 4\left(\frac{1}{2}\right)^4 = \frac{1}{4}.$$

2)

$$P(Y = 5) = \frac{8!}{5!\,(8-5)!}\left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^{8-5} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2}\left(\frac{1}{2}\right)^8 = 56\left(\frac{1}{2}\right)^8 = \frac{7}{32}.$$

The probability of 1st event can be expressed as

$$P(X = 3) = \frac{1}{4} \cdot \frac{8}{8} = \frac{8}{32}.$$

Therefore, the first event is more probable than the second, because

$$P(X = 3) = \frac{8}{32} > P(Y = 5) = \frac{7}{32}.$$

Answer: A beats B exactly in 3 games out of 4.