Answer on Question #64848 – Math – Statistics and Probability

Question

In a University, 20% of all students are graduates and 80% are undergraduates. The probability that a graduate student is married is 0.5 and the probability that an undergraduate student is married is 0.1. One student is selected at random. What is the probability that

(i) he/she is married

(ii) the student is a graduate if he/she is found to be married?

Solution

Let A_1 denote the event that a randomly selected student is, for example, graduates. Then the event $A_2 = \overline{A_1}$ means that a randomly selected student is undergraduates. So according to the task, we have

$$P(A_1) = 20\% = 0.2, \ P(A_2) = 80\% = 0.8.$$

Further, let B denote the event that a randomly selected student is married. Then the conditional probabilities are equal

$$P(B|A_1) = 0.5, \qquad P(B|A_2) = 0.1.$$

(i) By the law of total probability, the probability that a randomly selected student is married is equal to

$$P(B) = P(A_1)P(B|A_1) + P(A_2)P(B|A_2) = 0.2 \cdot 0.5 + 0.8 \cdot 0.1 = 0.18.$$

(ii) We know that the event B has occurred, and we want to calculate the conditional probability of the event A_1. By Bayes' theorem, we have

$$P(A_1|B) = \frac{P(A_1) \cdot P(B|A_1)}{P(B)} = \frac{0.2 \cdot 0.5}{0.18} = \frac{5}{9} \approx 0.56.$$

Answer: **(i)** 0.18; **(ii)** 0.56.