Answer on Question #64696 – Math – Linear Algebra

Question

1) Solve and Check

$2a+b-3c-d= -2$
$a-b-c+3d= 0$
$3a+2b+c+5d= 6$
$a-c-d= 2$

Solution

Let's rewrite the system of equations in matrix form and solve it by Gaussian elimination

$$\begin{matrix} 2 & 1 & -3 & -1 & -2 \\ 1 & -1 & -1 & 3 & 0 \\ 3 & 2 & 1 & 5 & 6 \\ 1 & 0 & -1 & -1 & 2 \end{matrix}$$

Change the order of rows in the matrix. Set the fourth row to be the first one

$$\begin{matrix} 1 & 0 & -1 & -1 & 2 \\ 2 & 1 & -3 & -1 & -2 \\ 1 & -1 & -1 & 3 & 0 \\ 3 & 2 & 1 & 5 & 6 \end{matrix}$$

Subtracting row 1 from row 3 and finally multiplying by (-1) in the modified matrix

$$\begin{matrix} 1 & 0 & -1 & -1 & 2 \\ 2 & 1 & -3 & -1 & -2 \\ 0 & 1 & 0 & -4 & 2 \\ 3 & 2 & 1 & 5 & 6 \end{matrix}$$

Adding row 1, multiplied by -2, to row 2

$$\begin{matrix} 1 & 0 & -1 & -1 & 2 \\ 0 & 1 & -1 & 1 & -6 \\ 0 & 1 & 0 & -4 & 2 \\ 3 & 2 & 1 & 5 & 6 \end{matrix}$$

Interchanging the second and third rows

$$\begin{matrix} 1 & 0 & -1 & -1 & 2 \\ 0 & 1 & 0 & -4 & 2 \\ 0 & 1 & -1 & 1 & -6 \\ 3 & 2 & 1 & 5 & 6 \end{matrix}$$

Adding row 1, multiplied by -3, to row 4

$$\begin{matrix} 1 & 0 & -1 & -1 & 2 \\ 0 & 1 & 0 & -4 & 2 \\ 0 & 1 & -1 & 1 & -6 \\ 0 & 2 & 4 & 8 & 0 \end{matrix}$$

Dividing row 4 by 2

$$\begin{matrix} 1 & 0 & -1 & -1 & 2 \\ 0 & 1 & 0 & -4 & 2 \\ 0 & 1 & -1 & 1 & -6 \\ 0 & 1 & 2 & 4 & 0 \end{matrix}$$

Subtracting the second row from the third row

$$\begin{matrix} 1 & 0 & -1 & -1 & 2 \\ 0 & 1 & 0 & -4 & 2 \\ 0 & 0 & -1 & 5 & -8 \\ 0 & 1 & 2 & 4 & 0 \end{matrix}$$

Multiplying the third row by (-1);

$$\begin{matrix} 1 & 0 & -1 & -1 & 2 \\ 0 & 1 & 0 & -4 & 2 \\ 0 & 0 & 1 & -5 & 8 \\ 0 & 1 & 2 & 4 & 0 \end{matrix}$$

Subtracting row 2 from row 4

$$\begin{matrix} 1 & 0 & -1 & -1 & 2 \\ 0 & 1 & 0 & -4 & 2 \\ 0 & 0 & 1 & -5 & 8 \\ 0 & 0 & 2 & 8 & -2 \end{matrix}$$

Adding row 3, multiplied by -2, to row 4

$$\begin{matrix} 1 & 0 & -1 & -1 & 2 \\ 0 & 1 & 0 & -4 & 2 \\ 0 & 0 & 1 & -5 & 8 \\ 0 & 0 & 0 & 18 & -18 \end{matrix}$$

Dividing row 4 by 18

$$\begin{matrix} 1 & 0 & -1 & -1 & 2 \\ 0 & 1 & 0 & -4 & 2 \\ 0 & 0 & 1 & -5 & 8 \\ 0 & 0 & 0 & 1 & -1 \end{matrix}$$

Adding row 4, multiplied by 5, to row 4

$$\begin{matrix} 1 & 0 & -1 & -1 & 2 \\ 0 & 1 & 0 & -4 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -1 \end{matrix}$$

Adding row 3 to row 1

$$\begin{matrix} 1 & 0 & 0 & -1 & 5 \\ 0 & 1 & 0 & -4 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -1 \end{matrix}$$

Adding row 4, multiplied by 4, to row 2

$$\begin{matrix} 1 & 0 & 0 & -1 & 5 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -1 \end{matrix}$$

Adding row 4 to row 1

$$\begin{matrix} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -1 \end{matrix}$$

Hence

a = 4, b = -2, c = 3, d = -1.

Let us check. Substituting this solution into the equations of the system and perform the calculation:

$2\cdot 4 + (-2) - 3\cdot 3 - (-1) = 8 - 2 - 9 + 1 = -2$
$4 - (-2) - 3 + 3\cdot(-1) = 4 + 2 - 3 - 3 = 0$
$3\cdot 4 + 2\cdot(-2) + 3 + 5\cdot(-1) = 12 - 4 + 3 - 5 = 6$
$4 - 3 - (-1) = 4 - 3 + 1 = 2$

Verification is successful.

Answer:
a = 4, b = -2, c = 3, d = -1.

Question

2)

r+s+2t-u= -3
2r+3s+3t+u= 2
4r+2s-t+u= 5
s+2t+2u=7

Solution

Let's rewrite the system of equations in the matrix form and solve it by the Gaussian elimination

$$\begin{matrix} 1 & 1 & 2 & -1 & -3 \\ 2 & 3 & 3 & 1 & 2 \\ 4 & 2 & -1 & 1 & 5 \\ 0 & 1 & 2 & 2 & 7 \end{matrix}$$

Subtracting row 1, multiplied by 2, from row 2;

$$\begin{matrix} 1 & 1 & 2 & -1 & -3 \\ 0 & 1 & -1 & 3 & 8 \\ 4 & 2 & -1 & 1 & 5 \\ 0 & 1 & 2 & 2 & 7 \end{matrix}$$

Subtracting row 1, multiplied by 4, from row 3

$$\begin{matrix} 1 & 1 & 2 & -1 & -3 \\ 0 & 1 & -1 & 3 & 8 \\ 0 & -2 & -9 & 5 & 17 \\ 0 & 1 & 2 & 2 & 7 \end{matrix}$$

Adding row 2, multiplied by 2, to row 3

$$\begin{matrix} 1 & 1 & 2 & -1 & -3 \\ 0 & 1 & -1 & 3 & 8 \\ 0 & 0 & -11 & 11 & 33 \\ 0 & 1 & 2 & 2 & 7 \end{matrix}$$

Subtracting row 2 from row 4

$$\begin{matrix} 1 & 1 & 2 & -1 & -3 \\ 0 & 1 & -1 & 3 & 8 \\ 0 & 0 & -11 & 11 & 33 \\ 0 & 0 & 3 & -1 & -1 \end{matrix}$$

Dividing row 3 by -11

$$\begin{matrix} 1 & 1 & 2 & -1 & -3 \\ 0 & 1 & -1 & 3 & 8 \\ 0 & 0 & 1 & -1 & -3 \\ 0 & 0 & 3 & -1 & -1 \end{matrix}$$

Subtracting row 3, multiplied by 3, from row 4

$$\begin{matrix} 1 & 1 & 2 & -1 & -3 \\ 0 & 1 & -1 & 3 & 8 \\ 0 & 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & 2 & 8 \end{matrix}$$

Dividing row 4 by 2

$$\begin{matrix} 1 & 1 & 2 & -1 & -3 \\ 0 & 1 & -1 & 3 & 8 \\ 0 & 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & 1 & 4 \end{matrix}$$

Adding row 4 to row 3

$$\begin{matrix} 1 & 1 & 2 & -1 & -3 \\ 0 & 1 & -1 & 3 & 8 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 4 \end{matrix}$$

Subtracting row 4, multiplied by 3, from row 2

$$\begin{matrix} 1 & 1 & 2 & -1 & -3 \\ 0 & 1 & -1 & 0 & -4 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 4 \end{matrix}$$

Adding row 4 to row 1

$$\begin{matrix} 1 & 1 & 2 & 0 & 1 \\ 0 & 1 & -1 & 0 & -4 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 4 \end{matrix}$$

Adding row 3 to row 2

$$\begin{matrix} 1 & 1 & 2 & 0 & 1 \\ 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 4 \end{matrix}$$

Subtract row 3, multiplied by 2, from row 1

$$\begin{matrix} 1 & 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 4 \end{matrix}$$

Subtracting row 2 from row 1

$$\begin{matrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 4 \end{matrix}$$

Hence

r = 2, s = -3, t = 1, u = 4

Let us check. Substituting this solution into the equations of the system and perform the calculation:

$2 + (-3) + 2 \cdot 1 - 4 = 2 - 3 + 2 - 4 = -3$
$2 \cdot 2 + 3 \cdot (-3) + 3 \cdot 1 + 4 = 4 - 9 + 3 + 4 = 2$
$4 \cdot 2 + 2 \cdot (-3) - 1 + 4 = 8 - 6 - 1 + 4 = 5$
$(-3) + 2 \cdot 1 + 2 \cdot 4 = -3 + 2 + 8 = 7$

Verification is successful.

Answer:
r = 2, s = -3, t = 1, u = 4.