Answer on Question #64522 – Math – Real Analysis

Question

Give an example of two divergence sequences X and Y such that: **(a)** their sum X+Y converges, **(b)** their product XY converges

Solution

Let's consider the sequences $x_n = \begin{cases} 0, & n-odd \\ 1, & n-even \end{cases}$ and $y_n = \begin{cases} 1, & n-odd \\ 0, & n-even \end{cases}$.

Since the subsequences $x_{2k} = 1 \underset{k\to\infty}{\to} 1$ and $x_{2k-1} = 0 \underset{k\to\infty}{\to} 0$ have different limit points, then the sequence $x_n = \begin{cases} 0, & n-odd \\ 1, & n-even \end{cases}$ is divergent.

Since the subsequences $y_{2k-1} = 1 \underset{k\to\infty}{\to} 1$ and $y_{2k} = 0 \underset{k\to\infty}{\to} 0$ have different limit points, then the sequence $y_n = \begin{cases} 1, & n-odd \\ 0, & n-even \end{cases}$ is divergent.

PART(a). Now we can consider the sum of those sequences: $x_n + y_n = \begin{cases} 1, & n-odd \\ 1, & n-even \end{cases} = 1 \underset{n\to\infty}{\to} 1$, hence the sequence $(x_n + y_n)$ is convergent.

PART(b). Now we can consider the product of those sequences: $x_n \cdot y_n = \begin{cases} 0, & n-odd \\ 0, & n-even \end{cases} = 0 \underset{n\to\infty}{\to} 0$, hence the sequence $(x_n \cdot y_n)$ is convergent.

Answer: $x_n = \begin{cases} 0, & n-odd \\ 1, & n-even \end{cases}$, $y_n = \begin{cases} 1, & n-odd \\ 0, & n-even \end{cases}$.