Answer on Question #64367 – Math – Abstract Algebra

Question

There exists a non-cyclic group in which every proper subgroup is cyclic. True or False. Prove.

Solution

Consider the dihedral group D_3, that is the symmetry group of an equilateral triangle. The multiplication table of this group is given below:

*	r_0	r_1	r_2	s_1	s_2	s_3
r_0	r_0	r_1	r_2	s_1	s_2	s_3
r_1	r_1	r_2	r_0	s_2	s_3	s_1
r_2	r_2	r_0	r_1	s_3	s_1	s_2
s_1	s_1	s_3	s_2	r_0	r_2	r_1
s_2	s_2	s_1	s_3	r_1	r_0	r_2
s_3	s_3	s_2	s_1	r_2	r_1	r_0

From the table we see that G is non-abelian, because the table is not symmetric. Therefore, G is non-cyclic. The proper nonempty subsets, which contain r_0 and closed under multiplication, are:

$$S_1 = \{r_0\},$$
$$S_2 = \{r_0, r_1, r_2\},$$
$$S_3 = \{r_0, s_1\},$$
$$S_4 = \{r_0, s_2\},$$
$$S_5 = \{r_0, s_3\}$$

The subsets S_1, S_3, S_4 and S_5 are cyclic subgroups because the elements r_0, s_1, s_2 and s_3 are their own inverses. The subset S_2 is a cyclic subgroup because r_1 is a generator for it.

Answer: True.