

Answer on Question #64249 – Math – Calculus

Question

Find the Taylor's expansion of $f(x)=\sin x$ at $x=0$.

Solution

$$x_0 = 0 \rightarrow (x - x_0) = x - 0 = x.$$

We arrange computations in two columns as follows:

$$\begin{array}{ll} f(x) = \sin x & f(0) = 0 \\ f'(x) = \cos x & f'(0) = 1 \\ f''(x) = -\sin x & f''(0) = 0 \\ f'''(x) = -\cos x & f'''(0) = -1 \\ f^{(4)}(x) = \sin x & f^{(4)}(0) = 0 \end{array}$$

Since the derivatives repeat in a cycle of four, we can write the Taylor's expansion as follows:

$$\begin{aligned} f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 + \dots \\ = 0 + \frac{1}{1!}x + \frac{0}{2!}x^2 + \frac{-1}{3!}x^3 + \frac{0}{4!}x^4 + \dots = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \\ = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \end{aligned}$$

The Taylor's expansion of $f(x)=\sin x$ at $x=0$ is

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}.$$

Answer: $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}.$