Answer on Question #64225 – Math – Calculus

Question 1

For the following transfer function:

$$H(s) = \frac{s^2 - 1}{s^2 + s + 1}$$

Plot the poles and zeros in the s-plane and determine and sketch the inverse Laplace transform $h(t)$

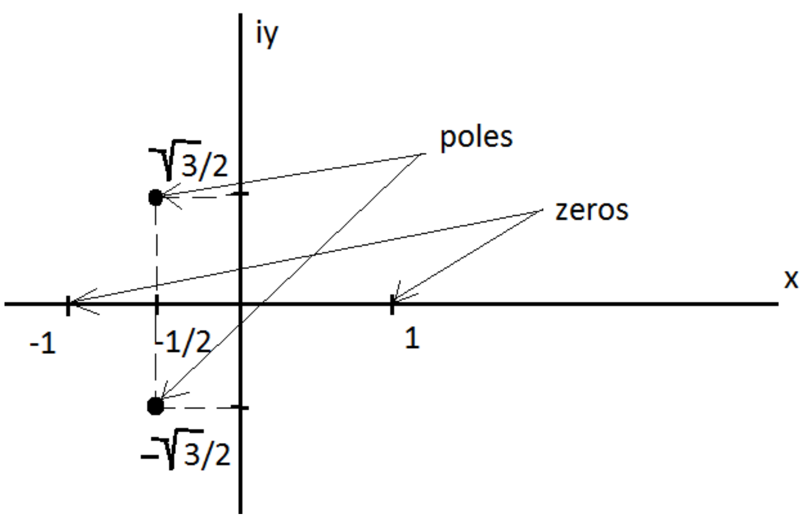
Solution

$$zeros: s_1 = 1 \; ; \; s_1 = -1$$

$$s^2 + s + 1 = 0$$

$$s = \frac{-1 \pm \sqrt{1 - 4}}{2}$$

$$poles: s_1 = \frac{-1 - i\sqrt{3}}{2} \; ; \; s_2 = \frac{-1 + i\sqrt{3}}{2}$$



$$H(s) = \frac{s^2 - 1}{s^2 + s + 1} = \frac{s^2 - 1 + s - s + 2 - 2}{s^2 + s + 1} = 1 - \frac{s + 2}{s^2 + s + 1}$$

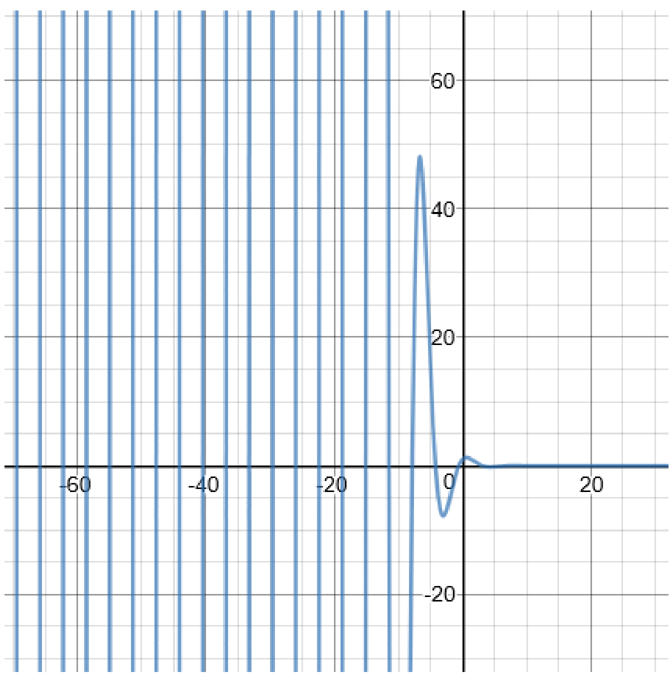
$$\lim_{s \to \frac{-1-i\sqrt{3}}{2}} \left(s + \frac{1+i\sqrt{3}}{2}\right) \frac{e^{st}(s+2)}{\left(s + \frac{1+i\sqrt{3}}{2}\right)\left(s - \frac{-1+i\sqrt{3}}{2}\right)} =$$

$$= \frac{\frac{-1-i\sqrt{3}}{2} + 2}{\frac{-1-i\sqrt{3}}{2} - \frac{-1+i\sqrt{3}}{2}} e^{\frac{-1-i\sqrt{3}}{2}t} = -\frac{3-i\sqrt{3}}{2i\sqrt{3}} e^{\frac{-1-i\sqrt{3}}{2}t}$$

$$h(t) = \delta(t) + res_1 + res_2$$

$$h(t) = \delta(t) + 2 \cdot \frac{e^{-\frac{t}{2}}}{2\sqrt{3}}\left(\sqrt{3}\cos\left(\frac{\sqrt{3}}{2}t\right) + 3\sin\left(\frac{\sqrt{3}}{2}t\right)\right)$$

$$h(t) = \delta(t) + e^{-\frac{t}{2}}\left(\cos\left(\frac{\sqrt{3}}{2}t\right) + \sqrt{3}\sin\left(\frac{\sqrt{3}}{2}t\right)\right)$$



Question 2

For the following transfer function:

$$H(s) = 3 + \frac{s+4}{s^2 + 3s + 2}$$

Find and sketch the inverse Laplace transform $h(t)$

Solution

$$s^2 + 3s + 2 = 0$$

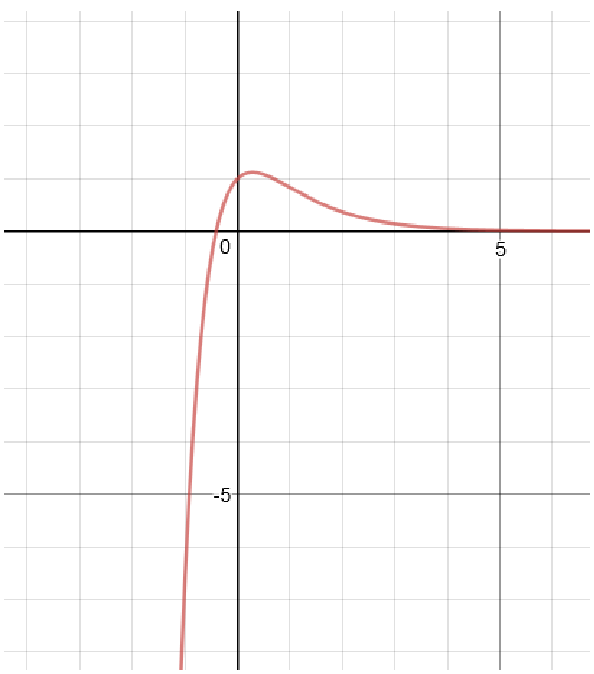
$$s = \frac{-3 \pm \sqrt{9-8}}{2}$$

$$poles: s_1 = -1\ ;\ s_2 = -2$$

$$res_1 = \lim_{s \to -1} (s+1) \frac{e^{st}(s+4)}{(s+1)(s+2)} = 3e^{-t}$$

$$res_2 = \lim_{s \to -2} (s+2) \frac{e^{st}(s+4)}{(s+1)(s+2)} = -2e^{-2t}$$

$$h(t) = 3\delta(t) + 3e^{-t} - 2e^{-2t}$$



Question 3

Solve the following second order differential equation:

$$y'' + 4y' + 3y = 4e^{-3t} \;\; ; \quad y(0) = 1 \, ; y'(0) = -1$$

and sketch the output $y(x)$

Solution

$$y(t) \to Y(p)$$

$$y'(t) \to pY(p) - y(0) = pY(p) - 1$$

$$y''(t) \to p^2Y(p) - p \cdot y(0) - y'(0) = p^2Y(p) - p + 1$$

$$4e^{-3t} \to \frac{4}{p+3}$$

$$p^2Y(p) - p + 1 + 4 \cdot (pY(p) - 1) + 3Y(p) = \frac{4}{p+3}$$

$$(p^2 + 4p + 3)Y(p) = \frac{4}{p+3} + p - 1 + 4$$

$$(p^2 + 4p + 3)Y(p) = \frac{4 + (p+3)^2}{p+3}$$

$$(p+1)(p+3)Y(p) = \frac{p^2 + 6p + 13}{p+3}$$

$$Y(p) = \frac{p^2 + 6p + 13}{(p+1)(p+3)^2}$$

$$\frac{A}{p+1} + \frac{B}{p+3} + \frac{C}{(p+3)^2} = \frac{p^2 + 6p + 13}{(p+1)(p+3)^2}$$

$$A(p+3)^2 + B(p+1)(p+3) + C(p+1) = p^2 + 6p + 13$$

$$A(p^2 + 6p + 9) + B(p^2 + 4p + 3) + Cp + C = p^2 + 6p + 13$$

$$\begin{cases} A + B = 1 \\ 6A + 4B + C = 6 \\ 9A + 3B + C = 13 \end{cases}$$

$$\begin{cases} A + B = 1 \\ 3A - B = 7 \end{cases}$$

$$A = 2\,; B = -1$$

$$C = 6 - 12 + 4 = -2$$

$$Y(p) = \frac{2}{p+1} - \frac{1}{p+3} - \frac{2}{(p+3)^2}$$

$$Y(p) \to y(t)$$

$$\frac{2}{p+1} \to 2e^{-t}\,;\ \frac{1}{p+3} \to e^{-3t}\,;\ \frac{2}{(p+3)^2} \to 2te^{-3t}$$

$$y(t) = 2e^{-t} - e^{-3t} - 2te^{-3t}$$