

Answer on Question #64191 – Math – Differential Equations

Question

1. $y = e^{ax} \cos^3 x \sin^2 x$. Find $\frac{dy}{dx}$.

Solution

The derivative of the product of three functions is

$$(u \cdot v \cdot w)' = u' \cdot v \cdot w + u \cdot v' \cdot w + u \cdot v \cdot w'$$

If

$$y = e^{ax} \cos^3(x) \sin^2(x),$$

then

$$\begin{aligned} \frac{dy}{dx} &= (e^{ax})' \cos^3(x) \sin^2(x) + e^{ax} (\cos^3(x))' \sin^2(x) + e^{ax} \cos^3(x) (\sin^2(x))' = \\ &= ae^{ax} \cos^3(x) \sin^2(x) \\ &\quad + e^{ax} (3\cos^2(x) \cdot (-\sin(x))) \cdot \sin^2(x) + 2\sin(x) \cos(x) \cdot \cos^3(x) \\ &= \\ &= ae^{ax} \cos^3(x) \sin^2(x) - 3e^{ax} \cos^2(x) \sin^3(x) + 2e^{ax} \sin(x) \cdot \cos^4(x) = \\ &= e^{ax} \cos(x) \sin(x) (a \cos^2(x) \sin(x) - 3 \cos(x) \sin^2(x) + 2 \cos^3(x)) \end{aligned}$$

Answer:

$$\frac{dy}{dx} = e^{ax} \cos x \sin x (a \cos^2 x \sin x - 3 \cos x \sin^2 x + 2 \cos^3 x).$$