Answer on Question #64135 – Math – Calculus

I-ANALYTIC GEOMETRY/RECTILINEAR MOTION

Question

1. Find the tangent line as directed to the curve Y = X4 + 2X3 – 2X2 -3X + 3 perpendicular to the line: X – 3Y = 2.

Solution

$$x - 3y = 2 \to y = \frac{x}{3} - \frac{2}{3}.$$

The slope is $\frac{1}{3}$. If the tangent line is perpendicular to $x - 3y = 2$, then the slope of tangent line is -3.

Next,

$$\frac{dy}{dx} = \frac{d}{dx}(x^4 + 2x^3 - 2x^2 - 3x + 3) = 4x^3 + 6x^2 - 4x - 3.$$

So,

$$4x^3 + 6x^2 - 4x - 3 = -3,$$

$$4x^3 + 6x^2 - 4x = 0,$$

$$2x(2x^2 + 3x - 2) = 0,$$

$$x_1 = 0,$$

$$2x^2 + 3x - 2 = 0,$$

$$D = 3^2 - 4 \cdot 2 \cdot (-2) = 25,$$

$$x_2 = \frac{-3-5}{2 \cdot 2} = \frac{-8}{4} = -2,$$

$$x_3 = \frac{-3+5}{2 \cdot 2} = \frac{2}{4} = \frac{1}{2}.$$

Equation of the tangent line to $y = y(x)$ at point $(x_0, y(x_0))$ is

$$y - y(x_0) = y'(x_0) \cdot (x - x_0)$$

1. If $x_1 = 0$, then

$$y(0) = 0^4 + 2 \cdot 0^3 - 2 \cdot 0^2 - 3 \cdot 0 + 3 = 3,$$

$$y - 3 = -3(x - 0),$$

$$y = -3x + 3.$$

2. If $x_2 = -2$, then

$$y(-2) = (-2)^4 + 2 \cdot (-2)^3 - 2 \cdot (-2)^2 - 3 \cdot (-2) + 3 = 1$$

$$y - 1 = -3(x + 2)$$

$$y = -3x - 5$$

3. If $x_3 = \frac{1}{2}$, then

$$y\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^4 + 2 \cdot \left(\frac{1}{2}\right)^3 - 2 \cdot \left(\frac{1}{2}\right)^2 - 3 \cdot \frac{1}{2} + 3 = \frac{21}{16}$$

$$y - \frac{21}{16} = -3\left(x - \frac{1}{2}\right)$$

$$y = -3x + \frac{45}{16}.$$

Answer: $y = -3x + 3; y = -3x - 5; y = -3x + \frac{45}{16}.$

Question

2. Find the tangent of the line as directed to the curve Y = X4 + 4X3 – 8X2 + 3X + 70 with slope 3.

Solution

$$\frac{dy}{dx} = \frac{d}{dx}(x^4 + 4x^3 - 8x^2 + 3x + 70) = 4x^3 + 12x^2 - 16x + 3.$$

So

$$4x^3 + 12x^2 - 16x + 3 = 3,$$

$$4x^3 + 12x^2 - 16x = 0,$$

$$4x(x^2 + 3x - 4) = 0,$$

$$x_1 = 0,$$

$$x^2 + 3x - 4 = 0,$$

$$D = 3^2 - 4 \cdot (-4) = 25,$$

$$x_2 = \frac{-3-5}{2} = -4,$$

$$x_3 = \frac{-3+5}{2} = 1.$$

Equation of the tangent line to $y = y(x)$ at point $\left(x_0, y(x_0)\right)$ is

$$y - y(x_0) = y'(x_0) \cdot (x - x_0)$$

1. If $x_1 = 0$, then

$$y(0) = 0^4 + 4 \cdot 0^3 - 8 \cdot 0^2 + 3 \cdot 0 + 70 = 70,$$

$$y - 70 = 3(x - 0),$$

$$y = 3x + 70.$$

2. If $x_2 = -4$, then

$$y(-4) = (-4)^4 + 4 \cdot (-4)^3 - 8 \cdot (-4)^2 + 3 \cdot (-4) + 70 = -70,$$

$$y + 70 = 3(x + 4),$$

$$y = 3x - 58.$$

3. If $x_3 = 1$, then

$$y(1) = 1^4 + 4 \cdot 1^3 - 8 \cdot 1^2 + 3 \cdot 1 + 70 = 70,$$

$$y - 70 = 3(x - 1),$$

$$y = 3x + 67.$$

Answer: $y = 3x + 70$; $y = 3x - 58$; $y = 3x + 67$.

Question

3. Find the equation of the line tangent to the curve Y = 3X2 – 4X and parallel to the line X – 2Y + 6 = 0.

Solution

$$x - 2y + 6 = 0 \rightarrow y = \frac{x}{2} + 3$$

The slope is $\frac{1}{2}$.

$$\frac{dy}{dx} = \frac{d}{dx}(3x^2 - 4x) = 6x - 4,$$

$$6x - 4 = \frac{1}{2},$$

$$x = \frac{\frac{1}{2}+4}{6},$$

$$x = \frac{3}{4}.$$

Equation of the tangent line to $y = y(x)$ at point $(x_0, y(x_0))$ is

$$y - y(x_0) = y'(x_0) \cdot (x - x_0),$$

$$y\left(\frac{3}{4}\right) = 3 \cdot \left(\frac{3}{4}\right)^2 - 4 \cdot \frac{3}{4} = -\frac{21}{16},$$

$$y + \frac{21}{16} = \frac{1}{2}\left(x - \frac{3}{4}\right),$$

$$y = \frac{1}{2}x - \frac{27}{16}.$$

Answer: $y = \frac{1}{2}x - \frac{27}{16}$.

Question

4. Find the equation of each normal line to the curve Y= X3 – 4X that is parallel to the line X + 8Y -8 = 0.

Solution

$$x + 8y - 8 = 0 \rightarrow y = -\frac{1}{8}x + 1$$

The slope of a normal line is $-\frac{1}{8}$, then the slope of the tangent line is 8.

$$\frac{dy}{dx} = \frac{d}{dx}(x^3 - 4x) = 3x^2 - 4,$$

$$3x^2 - 4 = 8,$$

$$x = 2 \text{ or } x = -2.$$

1. If $x = 2$, then

$$y(2) = 2^3 - 4 \cdot 2 = 0,$$

$$y - 0 = -\frac{1}{8}(x - 2),$$

$$y = -\frac{1}{8}x + \frac{1}{4}.$$

2. If $x = -2$, then

$$y(-2) = (-2)^3 - 4 \cdot (-2) = 0,$$

$$y - 0 = -\frac{1}{8}(x + 2),$$

$$y = -\frac{1}{8}x - \frac{1}{4}.$$

Answer: $y = -\frac{1}{8}x + \frac{1}{4}$; $y = -\frac{1}{8}x - \frac{1}{4}$.

Question

5. A particle moves along a straight line according to the law: S = 132 + 10t – 6t2 + 3t3. Find:

a) velocity and acceleration at any time t?

b) Velocity at t = 2 and

c) Acceleration at t= 3.

Solution

a) The velocity is

$$v(t) = \frac{dS}{dt} = \frac{d}{dt}(132 + 10\mathrm{t} - 6\mathrm{t}^2 + 3\mathrm{t}^3) = 9t^2 - 12t + 10$$

The acceleration is

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}(9t^2 - 12t + 10) = 18t - 12$$

b) Velocity at $t = 2$ is

$$v(2) = 9 \cdot 2^2 - 12 \cdot 2 + 10 = 22$$

c) Acceleration at $t = 3$ is

$$a(3) = 18 \cdot 3 - 12 = 42.$$

Answer: a) $9t^2 - 12t + 10$; $18t - 12$; **b)** 22; **c)** 42.

II-MAXIMA AND MINIMA

Question

1. A box with a square base is to have an open top. The area of the material in the box is to be 100 in square. What should the dimensions be in order to make the volume as large as possible?

Solution

Volume is

$V = a^2 h,$

hence

$$dV = a^2 dh + 2ahda = 0 \rightarrow adh + 2hda = 0$$

Area is

$$A = 4ah + a^2 = 100\ in^2,$$

hence

$$dA = 4adh + 4hda + 2ada = 0 \rightarrow$$

$$2adh + 2hda + ada = 2(adh + 2hda) + ada - 2hda = 2 \cdot 0 + ada - 2hda = ada - 2hda = 0,$$

$$2hda - ada = 0 \ \rightarrow h = \frac{a}{2}.$$

Therefore

$$A = 100\ in^2 = 4ah + a^2 = 4a\frac{a}{2} + a^2 = 3a^2.$$

The dimensions:

$$a = \sqrt{\frac{A}{3}} = \frac{10}{\sqrt{3}} \approx \ 5.77\ in;$$

$$h = \frac{a}{2} = \frac{5}{\sqrt{3}} \approx \ 2.89\ in.$$

Answer: $\frac{10}{\sqrt{3}}, \frac{5}{\sqrt{3}}.$