Question

If

$$y = e^{ax}\cos^3(x)\sin^2(x),$$

find $\dfrac{dy}{dx}$.

Solution

The derivative of the product of three functions is

$$(u \cdot v \cdot w)' = u' \cdot v \cdot w + u \cdot v' \cdot w + u \cdot v \cdot w'$$

If

$$y = e^{ax}\cos^3(x)\sin^2(x),$$

then

$$\frac{dy}{dx} = (e^{ax})'\cos^3(x)\sin^2(x) + e^{ax}(\cos^3(x))'\sin^2(x) + e^{ax}\cos^3(x)(\sin^2(x))' =$$

$$= ae^{ax}\cos^3(x)\sin^2(x)$$
$$+ e^{ax}(3\cos^2(x) \cdot (-\sin(x)) \cdot \sin^2(x) + 2\sin(x)\cos(x) \cdot \cos^3(x)) =$$

$$= ae^{ax}\cos^3(x)\sin^2(x) - 3e^{ax}\cos^2(x)\sin^3(x) + 2e^{ax}\sin(x) \cdot \cos^4(x).$$

Answer: $\dfrac{dy}{dx} = ae^{ax}\cos^3(x)\sin^2(x) - 3e^{ax}\cos^2(x)\sin^3(x) + 2e^{ax}\sin(x) \cdot \cos^4(x).$