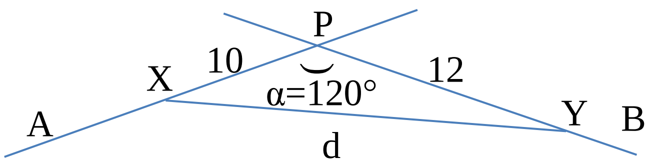
Answer on Question #64133 – Math – Calculus

Question

Two roads intersect at point P at an angle of 120°, as shown in the figure. Car X is driving from P towards A, and car Y is driving from P towards B. At a particular time, car X is 10 kilometers from P and traveling at 60 km/hr, while car Y is 12 kilometers from P and traveling at 80 km/hr. How fast is the distance between the two cars changing?

Solution



Let PX=*x*, PY=*y*, XY=*d*.

Applying the Law of Cosines to the triangle ΔPXY we can find the distance *d* between the cars X, Y:

$$d^2 = x^2 + y^2 - 2xy\cos 120° = x^2 + y^2 - 2xy\left(-\frac{1}{2}\right) = x^2 + y^2 + xy,$$

hence

$$d = \sqrt{x^2 + y^2 + xy}.$$

The distance between the two cars is changing at the rate which is equal to the derivative of d with respect to time t:

$$d'_t = \left(\sqrt{x^2 + y^2 + xy}\right)'_t = \frac{(x^2 + y^2 + xy)'_t}{2\sqrt{x^2 + y^2 + xy}} = \frac{2x \cdot x'_t + 2y \cdot y'_t + x'_t y + x y'_t}{2\sqrt{x^2 + y^2 + xy}}$$

Here we used the chain rule, the sum rule and the product rule, x'_t is the velocity of the X car, y'_t is the velocity of the Y car.

Plug the given values into the obtained expression:

$$d'_t = \frac{2x \cdot x'_t + 2y \cdot y'_t + x'_t y + x y'_t}{2\sqrt{x^2 + y^2 + xy}} = \frac{2 \cdot 10 \cdot 60 + 2 \cdot 12 \cdot 80 + 60 \cdot 12 + 10 \cdot 80}{2\sqrt{10^2 + 12^2 + 10 \cdot 12}} = \frac{4640}{38.1576} \approx 121.6 km/hr$$

Answer: the distance between the two cars is changing at the rate of $121.6\ km/hr$