Answer on Question #64131 – Math – Calculus

Question

I have the same question. A company manufacturers and sells x electronic drills per month. The monthly cost and price-demand equations are C(x)=72,000+40x and p(x)=300-x/20, 0≤x≤6000, respectively.

(A) Find the maximum revenue.

(B) Find the maximum profit, the production level that will realize the maximum profit, and the price the company should charge for each television set.

(C) If the government decides to tax the company $5 for each set it produces, how many sets should the company manufacture each month to maximize its profit? What is the maximum profit? What should the company charge for each set?

Solution

(A) Revenue is

$$R(x) = xp(x) = 300x - \frac{x^2}{20}.$$

To maximize $R(x)$, compute

$$R'(x) = \left(300x - \frac{x^2}{20}\right)' = 300 - \frac{x}{10};$$

$$R'(x) = 0 \Rightarrow 300 - \frac{x}{10} = 0;$$

$x = 3000.$

Find the second derivative of $R(x)$:

$$R''(x) = \left(300 - \frac{x}{10}\right)' = -\frac{1}{10} < 0 \Rightarrow \text{maximum of } R(x) \text{ is attained at}$$

$x = 3000.$

Maximum revenue is

$$R(3000) = 300 \cdot 3000 - \frac{3000^2}{20} = 450000(dollars)$$

(B) Profit is

$$P(x) = R(x) - C(x) = 300x - \frac{x^2}{20} - 72000 - 40x = 260x - \frac{x^2}{20} - 72000.$$

To maximize $P(x)$, compute

$$P'(x) = \left(260x - \frac{x^2}{20} - 72000\right)' = 260 - \frac{x}{10};$$

$$P'(x) = 0 \Rightarrow 260 - \frac{x}{10} = 0;$$

$x = 2600.$

Find the second derivative of $P(x)$:

$P''(x) = \left(260 - \frac{x}{10}\right)' = -\frac{1}{10} < 0 \Rightarrow$ maximum of $P(x)$ is attained at $x = 2600$, hence the production level that will realize the maximum profit is 2600.
Maximum profit is

$P(2600) = 260 \cdot 2600 - \frac{2600^2}{20} - 72000 = 266000(dollars)$

The price the company should charge for each television set is

$$p(2600) = 300 - \frac{2600}{20} = 170\ (dollars)$$

(C) New cost

$$C(x) = 72000 + 40x + 5x = 72000 + 45x$$

Profit

$$P(x) = R(x) - C(x) = 300x - \frac{x^2}{20} - 72000 - 45x = 255x - \frac{x^2}{20} - 72000.$$

To maximize $P(x)$, compute

$P'(x) = \left(255x - \frac{x^2}{20} - 72000\right)' = 255 - \frac{x}{10}.$

$P'(x) = 0 \Rightarrow 255 - \frac{x}{10} = 0;$

$x = 2550.$

Find the second derivative of $P(x)$:

$P''(x) = \left(255 - \frac{x}{10}\right)' = -\frac{1}{10} < 0 \Rightarrow$ maximum of $P(x)$ is attained at $x = 2550$, hence 2550 sets should the company manufacture each month to maximize its profit.
Maximum profit is

$P(2600) = 255 \cdot 2550 - \frac{2550^2}{20} - 72000 = 253125(dollars)$

The company should charge

$$p(2550) = 300 - \frac{2550}{20} = 172.5\ (dollars)$$

for each set.
When each set is taxed at $5, the maximum profit is $253125 when 2550 set are manufactured and sold for $172.5 each (difference is 172.5-170=$2.5).

Answer: (A) 450000; **(B)** 266000; 2600; 170; **(C)** 2550; 253125; 172.5.