Answer on Question #64114 – Math – Calculus

Question

2. For the function f(x) = 2x -arcsin x,
a) Locate all the local extrema and classify them as maxima or minima.
b) Are these local extrema also global extrema? Explain your answer.

Solution

$$f(x) = 2x - \arcsin x$$

Domain: [-1, 1]
a) Find the first derivative

$$f'(x) = (2x - \arcsin x)' = 2 - \frac{1}{\sqrt{1 - x^2}}$$

Derivative does not exist at $x = -1,$ and $x = 1$. These points are endpoints of the interval [-1, 1].

$f'(x) = 0 => \ 2 - \dfrac{1}{\sqrt{1 - x^2}} = 0;$

$x = -\dfrac{\sqrt{3}}{2}$ or $x = \dfrac{\sqrt{3}}{2}$.

Use the First Derivative Test

$f'(x) < 0$ on an open interval to the left of $x = -\sqrt{3}/2$ and $f'(x) > 0$ on an open interval to the right of $x = -\sqrt{3}/2$.

Therefore $f(x)$ has a relative minimum at $x = -\sqrt{3}/2$.

$f'(x) > 0$ on an open interval to the left of $x = \sqrt{3}/2$ and $f'(x) < 0$ on an open interval to the right of $x = \sqrt{3}/2$.

Therefore $f(x)$ has a relative maximum at $x = \sqrt{3}/2$.

b) We have that

$f(-1) = 2(-1) - \arcsin(-1) = -2 + \dfrac{\pi}{2};$

$f(1) = 2(1) - \arcsin(1) = 2 - \dfrac{\pi}{2};$

$f\left(-\dfrac{\sqrt{3}}{2}\right) = 2\left(-\dfrac{\sqrt{3}}{2}\right) - \arcsin\left(-\dfrac{\sqrt{3}}{2}\right) = -\sqrt{3} + \dfrac{\pi}{3};$

$f\left(\dfrac{\sqrt{3}}{2}\right) = 2\left(\dfrac{\sqrt{3}}{2}\right) - \arcsin\left(\dfrac{\sqrt{3}}{2}\right) = \sqrt{3} - \dfrac{\pi}{3}.$

$-\sqrt{3} + \dfrac{\pi}{3} < -2 + \dfrac{\pi}{2} < 2 - \dfrac{\pi}{2} < \sqrt{3} - \dfrac{\pi}{3}.$

Therefore $f(x)$ has a global minimum at $x = -\sqrt{3}/2$ and has a global maximum at $x = \sqrt{3}/2$.

Answer: a) $\left(-\frac{\sqrt{3}}{2}, -\sqrt{3}+\frac{\pi}{3}\right)$ is a local minimum, $\left(\frac{\sqrt{3}}{2}, \sqrt{3}-\frac{\pi}{3}\right)$ is a local maximum.
b) These local extrema are also global extrema.

Question

4. For the curve $y = \ln(1+x^2)$, find
a) intervals of increase and decrease and local extrema,
b) intervals of concavity and inflection points, then use this information to
c) sketch the graph.

Solution

$$y = \ln(1+x^2)$$

Domain: (-∞, ∞).
a) Find the first derivative

$$y' = (\ln(1+x^2))' = \frac{2x}{1+x^2}.$$

$y' = 0 => \dfrac{2x}{1+x^2} = 0 \;=>\; x = 0.$
Use the First Derivative Test
$y' < 0$ on an open interval to the left of $x = 0$ and $y' > 0$ on an open interval to the right of $x = 0$.
Therefore $f(x)$ has a relative minimum at $x = 0$.
$y(0) = \ln(1+0) = 0.$
y decreases on (-∞, 0);
y increases on (0, ∞).
b) Find the second derivative

$$y'' = \left(\frac{2x}{1+x^2}\right)' = 2\frac{1+x^2-2x^2}{(1+x^2)^2} = 2\frac{1-x^2}{(1+x^2)^2}.$$

$y' = 0 => 2\dfrac{1-x^2}{(1+x^2)^2} = 0 \;=> x = -1$ or $x = 1.$

$y'' < 0$ on an open interval to the left of $x = -1$ and $y'' > 0$ on an open interval to the right of $x = -1$. Therefore $(-1, \ln 2)$ is the inflection point.
$y'' > 0$ on an open interval to the left of $x = 1$ and $y'' < 0$ on an open interval to the right of $x = 1$. Therefore $(1, \ln 2)$ is the inflection point.
y is concave down on (-∞, -1) U (1, ∞).
y is concave up on (-1, 1).
c) The function is even:

$$y(x) = y(-x)$$

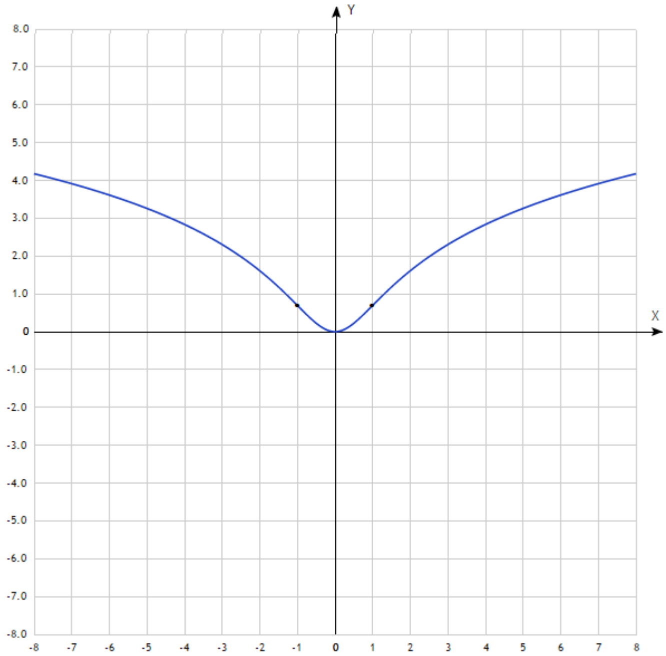

Fig. The graph of the curve $y = \ln(1 + x^2)$.

Answer:

a) interval of increase is (0, ∞), interval of decrease is (-∞, 0), local minimum is $(0, 0)$.

b) interval of concavity down is (-∞, -1) U (1, ∞); interval of concavity up is (-1, 1); the inflection points are $(-1, \ln 2)$, $(1, \ln 2)$.