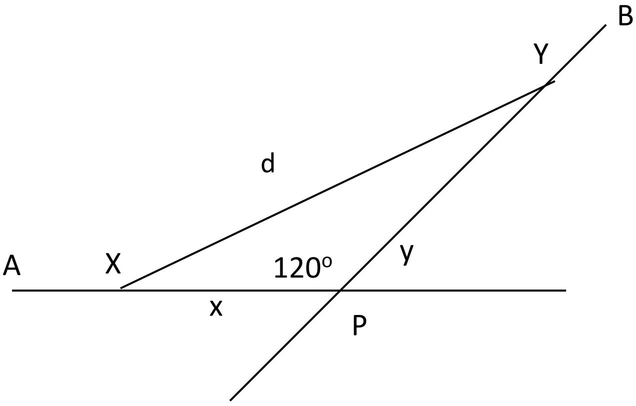
Answer on Question #64112 – Math – Calculus

Question

Two roads intersect at point P at an angle of 120 , as shown in the figure. Car X is driving from P towards A, and car Y is driving from P towards B. At a particular time, car X is 10 kilometers from P and traveling at 60 km/hr, while car Y is 12 kilometers from P and traveling at 80 km/hr. How fast is the distance between the two cars changing?

Solution

To find the rate of distance changing between two cars it is necessary to determine the distance between them, and then calculate the derivative of the distance over time.



Assume that car X at a time *t* is at the point X and car Y is at the point Y. The distance *d* between them can be found using the Law of Cosines applied to the triangle ΔPXY:

$$d = \sqrt{x^2 + y^2 - 2xy\cos 120°} = \sqrt{x^2 + y^2 - 2xy\left(-\frac{1}{2}\right)} = \sqrt{x^2 + y^2 + xy}$$

To find the derivative of the distance d with respect to time t we use the chain rule, the sum rule and the product rule:

$$d_t = \left(\sqrt{x^2 + y^2 + xy}\right)_t = \frac{(x^2+y^2+xy)_t}{2\sqrt{x^2+y^2+xy}} = \frac{2x \cdot x_t + 2y \cdot y_t + x_t y + x y_t}{2\sqrt{x^2+y^2+xy}} = \frac{2x \cdot x_t + 2y \cdot y_t + x_t y + x y_t}{2\sqrt{x^2+y^2+xy}}$$

Here x_t is the velocity of the X car, y_t is the velocity of the Y car. Plug $x = 10$, $x_t = 60$, $y = 12$, $y_t = 80$ into the formula for d_t:

$$d_t = \frac{2 \cdot 10 \cdot 60 + 2 \cdot 12 \cdot 80 + 60 \cdot 12 + 10 \cdot 80}{2\sqrt{10^2 + 12^2 + 10 \cdot 12}} = \frac{4640}{38.1576} \approx 121.6\ km/hr.$$

Answer: The distance between two cars is changed at the rate of $121.6\ km/hr$.