Answer on Question #64111 – Math – Calculus

Question

You are designing a rectangular poster which will have a rectangular printed area surrounded by margins of width 10 cm at the top and the bottom, and margins of width 7.5 cm at both sides. If the total area of the poster is to be 1m^2, find the maximum possible printed area.

Solution

The total area is

$$S_t = 1\ m^2.$$

If $a > 0$ is the width of the poster, then

$$l = \frac{S_t}{a} = \frac{1}{a} \text{ is the length of the poster.}$$

The printed area is

$$S_p = (a - 0.075 \cdot 2) \cdot \left(\frac{1}{a} - 0.1 \cdot 2\right) = (a - 0.15)\left(\frac{1}{a} - 0.2\right),$$

where $a > 0.15$ and $\frac{1}{a} > 0.2$.

Since $S_p = S_p(a)$ is a differentiable function on the entire interval [0.15, 5], the maximum of $S_p(a)$ can be attained at the endpoints or at the critical points.

The derivative of S_p is given by

$$S_p' = \left((a - 0.15) \cdot \left(\frac{1}{a} - 0.2\right)\right)' = (a - 0.15)'\left(\frac{1}{a} - 0.2\right) + (a - 0.15)\left(\frac{1}{a} - 0.2\right)'$$

$$= \frac{1}{a} - 0.2 - \frac{1}{a^2}(a - 0.15) = \frac{0.15}{a^2} - 0.2$$

If $S_p' = 0$, then the positive critical point is given by

$$a_1 = \sqrt{\frac{0.15}{0.2}} = \sqrt{0.75} = \frac{\sqrt{3}}{2}.$$

The derivative S_p' is positive on the left of the critical point and negative on the right of the critical point.

Thus, the maximum value of S_p is given by

$$\max S_p = S_p(a_1) = \left(\frac{\sqrt{3}}{2} - 0.15\right)\left(\frac{1}{\frac{\sqrt{3}}{2}} - 0.2\right) = \frac{\sqrt{3} - 0.3}{2} \cdot \left(\frac{2\sqrt{3} - 0.6}{3}\right) = \frac{\left(\sqrt{3} - 0.3\right)^2}{3} = \frac{3 - 0.6\sqrt{3} + 0.09}{3} =$$

$$= 1 - 0.2\sqrt{3} + 0.03 = 1.03 - 0.2\sqrt{3} \approx 0.6836.$$

Answer: $1.03 - 0.2\sqrt{3}$.