Answer on Question #64110 – Math – Calculus

Question

1. Suppose that for all x >= 0, the average value of f(x) on [0; x] is equal to x. What is the function f(x)?

Solution

The average value of f(x) on [0;x] is $\frac{\int_0^x f(u)du}{x}$.

Given the average value of f(x) is x, we get the following equation:

$$\frac{\int_0^x f(u)du}{x-0} = x$$

$$\int_0^x f(u)du = x^2 \text{ (1)}$$

Let F(x) be such a function that $\frac{dF(x)}{dx} = f(x)$

Applying the fundamental theorem of calculus (Newton-Leibniz formula)

$$\int_a^b f(u)du = F(b) - F(a) \text{ (2)}$$

to equation (1) we get the next equation:

$$F(x) - F(0) = x^2$$

$$\frac{d(F(x) - F(0))}{dx} = \frac{d(x^2)}{dx}$$

$$\frac{dF(x)}{dx} = f(x) = 2x$$

Answer: $f(x) = 2x$

Question

2. The equation x^4+x^3-4 = 0 has a solution near x = 1. Use Newton's method to find this solution correct to 8 significant figures.

Solution

Let $x_0 = 1$ and equation is $f(x) = 0$, where

$$f(x) = x^4 + x^3 - 4;$$

$$\frac{df(x)}{dx} = 4x^3 + 3x^2.$$

Iteration formula for this method is

$$x_{n+1} = x_n - \frac{f(x_n)}{\frac{df(x_n)}{dx}} = x_n - \frac{x_n^4 + x_n^3 - 4}{4x_n^3 + 3x_n^2},$$

where $x_0 = 1$.

Just use it while $|x_{n+1} - x_n| > epsilon = 10^{-8}$.

n	x_n
1	1.2857142857
2	1.2219746898
3	1.2173567366
4	1.2173336996
5	1.2173336991
6	1.2173336991

Then we have at the last step that

$$x_n = 1.217333699$$

When we round it to 8 digits after dot we will have

$$x_n = 1.21733370$$

Answer:

x = 1.21733370.

Question

3. At a certain moment, a triangle has base length 10 cm and perpendicular height 8 cm. The length is increasing at a rate of 1 cm/s while the height is decreasing at 1 cm/s. How fast is the area changing? Is the area increasing or decreasing?

Solution

The area of the triangle is a product of base length by the perpendicular height.

The area of the given triangle at the moment of time t:

$S(t) = \frac{1}{2}$(10 cm +1cm/s·t)(8 cm – 1 cm/s · t)

S_0=S(0)= 40 cm^2 is the initial area of square.

S(t) = $\frac{1}{2}$(80 cm^2 – 10 cm^2/s · t + 8 cm^2/s · t – (t* cm/s)^2).

S(t) = $\frac{1}{2}$(80 cm^2 – 8 cm^2/s · t – (t* cm/s)^2)=

=40 cm^2 – 4 cm^2/s · t $-\frac{1}{2}$(t* cm/s)^2, that is,

S(t) =40 – 4 · t $-\frac{1}{2}$t^2

We can see that it is a vertical parabola, open down because the coefficient of t^2 is $-\frac{1}{2} < 0.$

Coordinates of the vertex are

$t_{vert} = -\frac{-4}{2\cdot\left(-\frac{1}{2}\right)} = -4,$

$S_{vert} = S(t_{vert}) = 40 - 4\cdot(-4) - \frac{1}{2}\cdot(-4)^2 = 48.$

Time is nonnegative, hence $t \geq 0.$

So after t=0 S(t) will decrease from the value 40 cm^2 to zero with a passage of time, but geometrically the area of the triangle can't be less than zero.

The area is changing at the rate of

$\frac{d}{dt}\mathrm{S(t)} = \frac{d}{dt}\left(40 - 4 \cdot \mathrm{t} - \frac{1}{2}\mathrm{t\hat{}2}\right) = -4 - t < 0\ (\mathrm{cm\hat{}2/s})$

Answer:

The rate of change is $(-4-t)\ \mathrm{cm}^\wedge 2/s$

The area of triangle will be decreasing.