

Answer on Question #63951 – Math – Calculus

Question

A man is walking along a sidewalk at the rate of 5 feet/sec. A searchlight on the ground 30 feet from the walk is kept trained on him. At what rate is the searchlight revolving when the man is 20 feet away from the point on the sidewalk nearest the light?

Solution

$$\tan\theta = \frac{x}{30} \rightarrow \theta = \arctan\left(\frac{x}{30}\right) \rightarrow \frac{d\theta}{dt} = \frac{1}{1 + \left(\frac{x}{30}\right)^2} * \frac{d}{dt}\left(\frac{x}{30}\right) = \frac{30}{x^2 + 900} \cdot \frac{dx}{dt}$$

If $x = 20$ and $\frac{dx}{dt} = 5$, then

$$\frac{d\theta}{dt} = \frac{30}{20^2 + 900} \cdot 5 = \frac{30}{400 + 900} \cdot 5 = \frac{150}{1300} = \frac{3}{26} \approx 0.1154 \frac{\text{feet}}{\text{sec}}$$

Answer: $0.1154 \frac{\text{feet}}{\text{sec}}$.