

Answer on Question #63945 – Math – Calculus

Question

A closed box with a square base is to have a volume of 2000 cu inches. The material for the top and bottom of the box is to cost 3 pesos per square inch, and the material for the sides is to cost 1.50 pesos/square inch. If the cost of the material is to be least, find the dimensions of the box.

Solution

If the edge of the square base is x , then its area is x^2 , so the cost of the top and bottom is $2x^2$.

Since the volume is $V=2000 = hx^2$, then the height $h = 2000/x^2$.

The surface area is $SA = 4 \cdot (2000/x^2)x + 2x^2$,

The price of material is given by

$$y = 1.5 \cdot 4 \cdot (2000/x^2)x + 3 \cdot (2x^2),$$

$$y = 12000/x + 6x^2,$$

$$y = 12000x^{-1} + 6x^2,$$

The first derivative is

$$f'(x) = -12000x^{-2} + 12x,$$

If we set it equal to zero and solve, then we will get the value of x to form the box with the least cost:

$$-12000x^{-2} + 12x = 0,$$

$$12x^3 - 12000 = 0,$$

$$x^3 = 1000,$$

$$x = 10.$$

If $x = 10$, then $h = 2000/10^2 = 20$, so the box is 10" x 10" x 20".

Answer: 10 square inch, 10 square inch, 20 square inch.