Answer on Question #63934 – Math – Calculus

Question

Find the numbers whose sum is x, if the product of one by the square of the other is to be a maximum.

Solution

$$ab^2 \to max$$

$$a + b = x$$

It follows from the equation that $b = x - a$, then

$$ab^2 = a(x-a)^2$$

$$f(a) = a(x-a)^2 \to max$$

$$\frac{df}{da} = (a(x-a)^2)' = (x-a)^2 + a \cdot 2 \cdot (x-a) \cdot (-1) = (x-a) \cdot \big((x-a) - 2a\big) =$$

$$= (x-a)(x-3a),$$

$$\frac{df}{da} = (x-a)(x-3a)$$

$$\frac{df}{da} > 0 \iff$$

$$(x-a)(x-3a) > 0 \iff$$

$$\left[\begin{array}{l} \begin{cases} x-a > 0, \\ x-3a > 0; \end{cases} \\ \begin{cases} x-a < 0, \\ x-3a < 0\,; \end{cases} \end{array}\right. \iff$$

$$\left[\begin{array}{l} \begin{cases} a < x, \\ a < x/3\,; \end{cases} \\ \begin{cases} a > x, \\ a > \dfrac{x}{3}. \end{cases} \end{array}\right.$$

<u>Case 1</u>: $x \neq 0$.

If $a = 0$, then $b = x$ and $ab^2 = 0$. Obviously it is not a global maximum.

If $a > 0$, then $\frac{df}{da} > 0 <=> a > x$ or $a < \frac{x}{3}$.

If $a = x$, then $b = x - a = 0$, $ab^2 = 0$. Obviously it is not a global maximum.

If $a = \frac{x}{3}$, then $b = x - a = \frac{2x}{3}$, $\quad ab^2 = \frac{x}{3} \cdot \frac{4}{9}x^2 = \frac{4}{27}x^3$.

If $a < 0$, then $\frac{df}{da} > 0 <=> a > \frac{x}{3}$ or $a < x$, hence $a = \frac{x}{3}$ is a local maximum and

$b = x - a = x - \frac{x}{3} = \frac{2x}{3};\ ab^2 = \frac{x}{3} \cdot \frac{4x^2}{9} = \frac{4}{27}x^3$.

Case 2: $x = 0$.

If $x = 0$, then $b = -a$, $ab^2 = a(-a)^2 = a^3$ is increasing and the maximum is not attained.

Thus, the numbers are $\frac{x}{3}$ and $\frac{2x}{3}$ whose sum is x if the product of one by the square of the other is to be a maximum.

Answer: $\frac{x}{3}$ and $\frac{2x}{3}$**,** where $x \neq 0$.