Answer on Question #63932 – Math – Analytic Geometry

Question

Find an equation of each normal line to the curve: $y = x^3 - 4x$ that is parallel to the line $x + 8y - 8 = 0$.

Solution

If $y = x^3 - 4x$, then the derivative is equal to $y' = 3x^2 - 4$.

Let us rewrite the equation of the line $x + 8y - 8 = 0$ in the slope-intercept form:

$$y = 1 - \frac{x}{8}.$$

Its slope is

$$m = -\frac{1}{8}.$$

The slope of the required normal line is equal to

$$-\frac{1}{y'(x_0)} = -\frac{1}{3x_0^2 - 4}.$$

Since the normal line is parallel to the given line, then we conclude that their slopes are equal:

$$-\frac{1}{8} = -\frac{1}{3x_0^2 - 4}.$$

It follows from this formula that

$$3x_0^2 - 4 = 8 \Rightarrow 3x_0^2 = 12 \Rightarrow x_0^2 = 4 \Rightarrow \left[\begin{array}{l} x_0 = 2 \\ x_0 = -2 \end{array}\right..$$

Let us consider each of these cases.

If $x_0 = 2$ then the point on the curve is $(2, 0)$. Since the equation of the normal line is

$$\tilde{y} = -\frac{1}{y'(x_0)}(x - x_0) + y(x_0)$$

and

$$y'(x_0) = 3x^2 - 4|_{x=2} = 8;\ y(x_0) = 2^3 - 4 \cdot 2 = 0,$$

then the required normal line has the following equation:

$$y = -\frac{1}{8}(x - 2) \Leftrightarrow y = \frac{1}{4} - \frac{x}{8}.$$

If $x_0 = -2$ then the point on the curve is $(-2, 0)$. Since the equation of the normal line is

$$\tilde{y} = -\frac{1}{y'(x_0)}(x - x_0) + y(x_0)$$

and

$$y^{/}(x_0) = 3x^2 - 4|_{x=-2} = 8,\ y(x_0) = (-2)^3 - 4 \cdot (-2) = -8 + 8 = 0,$$

then the required normal line has the following equation:

$$y = -\frac{1}{8}(x+2) \Leftrightarrow y = -\frac{1}{4} - \frac{x}{8}.$$

Answer: $y = \frac{1}{4} - \frac{x}{8}$ and $y = -\frac{1}{4} - \frac{x}{8}$.