Answer on Question #63930 – Math – Calculus

Question

Find the tangent line to the curve

$$y = x^4 + 4x^3 - 8x^2 + 3x + 70$$

with slope 3.

Solution

If the slope of tangent line is equal to 3, then

$$y' = (x^4 + 4x^3 - 8x^2 + 3x + 70)' = 4x^3 + 12x^2 - 16x + 3 = 3$$

at this point. So

$$4x^3 + 12x^2 - 16x = 0,$$

$$4x(x^2 + 3x - 4) = 0,$$

$$x = 0 \text{ or } x^2 + 3x - 4 = 0\,.$$

Applying Viet's theorem the roots of $x^2 + 3x - 4 = 0$ are 1 and -4.

Thus, $x_1 = 0, x_2 = -4, x_3 = 1$.

The equation of the tangent line is

$$y - y(x_0) = y'(x_0)(x - x_0)$$

If $x_1 = 0$, then

$$y\,(x_1) = 0^4 + 4 \cdot 0^3 - 8 \cdot 0^2 + 3 \cdot 0 + 70 = 70$$

and tangent line at the point $(0{,}70)$ is

$$y - 70 = 3x,$$

$$y = 3x + 70.$$

If $x_2 = -4$ then

$$y\,(x_2) = (-4)^4 + 4 \cdot (-4)^3 - 8 \cdot (-4)^2 + 3 \cdot (-4) + 70 = -70$$

and tangent line at the point $(-4, -70)$ is

$$y + 70 = 3(x + 4),$$

$$y = 3x - 58.$$

If $x_3 = 1$, then

$$y\,(x_3) = 1^4 + 4 \cdot 1^3 - 8 \cdot 1^2 + 3 \cdot 1 + 70 = 70$$

and tangent line at the point $(1{,}70)$ is

$$y - 70 = 3(x - 1),$$

$$y = 3x + 67.$$

<u>Answer</u>:

$y = 3x + 70$ at $(0, 70)$,

$y = 3x - 58$ at $(-4, -70)$,

$y = 3x + 67$ at $(1, 70)$.