Answer on Question #63929 – Math – Calculus

Question

Find the tangent line to the curve y = x^4 + 2x^3 - 2x^2 - 3x + 3 perpendicular to the line x - 3y = 2

Solution

The slope of the line $x - 3y = 2$ is $k_1 = \frac{1}{3}$. The slope of the perpendicular line is

$$k_2 = -\frac{1}{k_1} = -3.$$

Using $y = x^4 + 2x^3 - 2x^2 - 3x + 3$ compute

$y' = (x^4 + 2x^3 - 2x^2 - 3x + 3)' = 4x^3 + 6x^2 - 4x - 3.$

Next,

$$y' = -3 \ \rightarrow \ 4x^3 + 6x^2 - 4x - 3 = -3 \rightarrow \ 4x^3 + 6x^2 - 4x = 0 \ \rightarrow$$

$$2x(2x^2 + 3x - 2) = 0 \rightarrow$$

$$\rightarrow \ \ x = 0, \ x = -2, \ x = \frac{1}{2} \rightarrow$$

$y(0) = 0^4 + 2 \cdot 0^3 - 2 \cdot 0^2 - 3 \cdot 0 + 3 = 3,$

$y(-2) = (-2)^4 + 2 \cdot (-2)^3 - 2 \cdot (-2)^2 - 3 \cdot (-2) + 3 = 1,$

$y\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^4 + 2 \cdot \left(\frac{1}{2}\right)^3 - 2 \cdot \left(\frac{1}{2}\right)^2 - 3 \cdot \left(\frac{1}{2}\right) + 3 = \frac{21}{16}.$

The equation of the tangent line is

$y - y(x_0) = y'(x_0) \cdot (x - x_0).$

The tangent line at the point $(0,3)$:

$y - 3 = -3x \ \rightarrow \ \ y = -3x + 3.$

The tangent line at the point $(-2,1)$:

$y - 1 = -3(x + 2) \rightarrow \ \ y = -3x - 5.$

Tangent line at the point $\left(\frac{1}{2}, \frac{21}{16}\right)$:

$y - \frac{21}{16} = -3(x - \frac{1}{2}) \ \rightarrow \ \ y = -3x + \frac{45}{16}.$

Answer: $y = -3x + 3; \ \ y = -3x - 5; \ \ y = -3x + \frac{45}{16}.$