

Answer on Question #63809 – Math – Statistics and Probability

Question

A researcher hypothesizes that people who listen to music via headphones have greater hearing loss and will thus score lower on a hearing test than those in the general population. On a standard hearing test $\mu=19.2$. The researcher gives this same test to a random sample of 12 individuals who regularly use headphones. Their scores on the test are 17,15,21,13,26,23,24,20,18,18,22,21. What is the t_{cv} ? compute t_{obt}

Solution

$$H_0: \mu = 19.2$$

$$H_1: \mu < 19.2$$

If scores are 17, 15, 21, 13, 26, 23, 24, 20, 18, 18, 22, 21, then

X	13	15	17	18	20	21	22	23	24	26
n	1	1	1	2	1	2	1	1	1	1

$$\bar{x} = \frac{13+15+17+2\cdot 18+20+2\cdot 21+22+23+24+26}{12} = 19.83;$$

$$\begin{aligned} \frac{\sum_{i=1}^N (X_i - \bar{x})^2}{N - 1} &= \\ &= \frac{(13 - 19.83)^2 + (15 - 19.83)^2 + (17 - 19.83)^2 + 2(18 - 19.83)^2}{11} + \\ &+ \frac{(20 - 19.83)^2 + 2(21 - 19.83)^2 + (22 - 19.83)^2 + (23 - 19.83)^2}{11} + \\ &+ \frac{(24 - 19.83)^2 + (26 - 19.83)^2}{11} \approx 14.333; \end{aligned}$$

$$s = \sqrt{\frac{\sum_{i=1}^N (X_i - \bar{x})^2}{N - 1}} = \sqrt{14.333} \approx 3.79.$$

If the significance level is $\alpha = 0.05$, then the critical value is

$$t_{cv} = -t_{0.05,11} = -1.796.$$

The value of test statistics is

$$t_{obt} = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{19.83 - 19.2}{3.79/\sqrt{12}} \approx 0.576.$$

Rejection region is $t_{obt} \leq t_{cv}$.

The value of the test statistic, $t_{obt} \approx 0.576$, does not lie in the rejection region, because $t_{obt} > t_{cv}$, so we cannot reject the null hypothesis at $\alpha = 0.05$.

Answer: $t_{cv} = -1.796$; $t_{obt} \approx 0.576$.