Answer on Question #63800 – Math – Analytic Geometry

Question

Tangent to the line 4x-3y=6 at (3,2) and passing through (2,-1).

Solution

The direction vector from (3,2) to (2,-1) is $\vec{a} = (3-2, 2-(-1)) = (1,3)$.
The vector $(4,-3)$ is orthogonal to the line $4x-3y=6$.
The vector $\vec{b} = (3,4)$ is orthogonal to the vector $(4,-3)$.
Consequently, $\vec{b} = (3,4)$ is the direction vector of the line $4x-3y=6$.

Let φ be the acute angle between the lines with the direction vectors $\vec{a}$ and $\vec{b}$.
Then $0 \le \varphi \le \frac{\pi}{2}$ and

$$\cos\varphi = \frac{|(\vec{a},\vec{b})|}{\sqrt{(\vec{a},\vec{a})}\sqrt{(\vec{b},\vec{b})}} = \frac{|1\cdot 3+3\cdot 4|}{\sqrt{1\cdot 1+3\cdot 3}\sqrt{3\cdot 3+4\cdot 4}} = \frac{3}{\sqrt{10}}.$$

Hence the required tangent is

$$\tan\varphi = \sqrt{\frac{1}{(\cos\varphi)^2}-1} = \sqrt{\frac{10}{9}-1} = \frac{1}{3}.$$

Answer: $\frac{1}{3}$.