Answer on Question #63773 – Math – Statistics and Probability

Question

The probability of a shooter hitting a target is 2/4. How many minimum number of times must he/she fire so that the probability of hitting the target at least once is more 0.99?

Solution

Let p denote the probability of a shooter hitting a target at each of n times. Then by the product rule for n independent events the probability of hitting the target n times is p^n while the probability of not hitting the target is $(1-p)^n$.
Finally, the probability of hitting the target at least once is $1-(1-p)^n$.
Therefore, we have

$$1-(1-1/2)^n > 0.99;$$
$$1-(1/2)^n > 0.99;$$
$$1-0.99 > (1/2)^n;$$
$$1/2^n < 0.01;$$
$$\frac{1}{0.01} < 2^n;$$
$$2^n > 100;$$
$$\log 2^n > \log 10^2;$$
$$n \cdot \log 2 > 2\log 10;$$
$$n > \frac{2}{\log 2};$$
$$n > 6.64.$$

Thus, the minimum number of times is $n = 7$.
Check:
$1-\frac{1}{2^6} \approx 0.984; \quad 1-\frac{1}{2^7} \approx 0.992.$

Answer: 7.