Answer on Question #63756 – Math – Calculus

Question

To fill an order for 100 units of a product, a firm wishes to distribute the production between its two plants, Plant 1 and Plant 2.

The total cost function is given by

$$c = f(q_1, q_2) = 0.5q_1^2 + 2q_1 + 32q_2 + 500,$$

where q_1 and q_2 are the number of units produced at Plants 1 and 2, respectively. How should the output be distributed in order to minimize costs?

Solution

We have that

$$q_1 + q_2 = 100 \implies q_2 = 100 - q_1.$$

Then

$c = 0.5q_1^2 + 2q_1 + 32q_2 + 500 = 0.5q_1^2 + 2q_1 + 32(100 - q_1) + 500;$

$c = g(q_1) = 0.5q_1^2 - 30q_1 + 3700.$

To minimize the function $g(q_1)$ we find

$g'(q_1) = (0.5q_1^2 - 30q_1 + 3700)' = 0.5 \cdot 2q_1 - 30 = q_1 - 30;$

$g'(q_1) = 0 \implies q_1 - 30 = 0 \implies q_1 = 30;$

$g''(q_1) = (g'(q_1))' = (q_1 - 30)' = 1 > 0 \implies q_1 = 30$ is minimum.

If $q_1 = 30$, then $q_2 = 100 - 30 = 70$ and

$c = f(30, 70) = 0.5 \cdot 30^2 + 2 \cdot 30 + 32 \cdot 70 + 500 = 3190.$

Answer: 30 and 70 units produced at Plants 1 and 2, respectively.