Answer on Question #63635 – Math – Analytic Geometry

Question

Let O be the origin and OA = a1i + a2j + a3k. Find the equation of the plane that contains vectors I + j + k and 2i + j + k

Solution

The equation of a plane is

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

where $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ is the normal vector to the plane, $P(x_0, y_0, z_0)$ is the point on the plane. Since the plane contains the vector $\vec{\imath} + \vec{\jmath} + \vec{k}$, then it passes through the origin and $(x_0, y_0, z_0) = (0,0,0)$.

We can use the cross product of two vectors $\vec{\imath} + \vec{\jmath} + \vec{k}$ and $2\vec{\imath} + \vec{\jmath} + \vec{k}$ as the normal vector to the plane since both of them are in the plane (any vector that is orthogonal to both of these will also be orthogonal to the plane):

$$\vec{n} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} \vec{\imath} - \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} \vec{\jmath} + \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} \vec{k} = (1 \cdot 1 - 1 \cdot 1)\vec{\imath} -$$

$$-(1 \cdot 1 - 2 \cdot 1)\vec{\jmath} + (1 \cdot 1 - 2 \cdot 1)\vec{k} = (1 - 1)\vec{\imath} - (1 - 2)\vec{\jmath} + (1 - 2)\vec{k} = 0\vec{\imath} + \vec{\jmath} -$$

$$-\vec{k}\ .$$

The equation of the plane is then

$$0 \cdot (x - 0) + 1 \cdot (y - 0) - 1 \cdot (z - 0) = 0;$$

$$y - z = 0\ .$$

Answer: the equation of the plane is $y - z = 0$.