Answer on Question #63634 – Math – Linear Algebra

Question

B = [0 a 0 0]
b 0 0 0
0 0 c 0
0 0 0 d

Let Bn the (n x n) submatrix in the TOP left hand corner of B. Define B1, B2, B3 and B4. Compute determinate of B1, B2 , B3 and B4. Find conditions of a, b, c, d such that 4 determinants cannot be negative.

Solution

$B1 = (0)$ is 1×1 matrix, $det(B1) = 0$ for all values of a, b, c, d.

$B2 = \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}$ is 2×2 matrix, $det(B2) = 0 \cdot 0 - b \cdot a = -ab$. So $det(B2) \geq 0$ is equivalent to $ab \leq 0$.

$B3 = \begin{pmatrix} 0 & a & 0 \\ b & 0 & 0 \\ 0 & 0 & c \end{pmatrix}$ is 3×3 matrix, $det(B3) = c \cdot \det(B2) = -abc$. So $det(B3) \geq 0$ is equivalent to $abc \leq 0$.

$B4 = \begin{pmatrix} 0 & a & 0 & 0 \\ b & 0 & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{pmatrix}$ is 4×4 matrix, $det(B4) = d \cdot det(B3) = -abcd$. So $det(B4) \geq 0$ is equivalent to $abcd \leq 0$.

These 4 determinants cannot be negative if one of the following conditions holds:

1) $ab = 0$;
2) $(ab < 0)$ and $(c = 0)$;
3) $(ab < 0)$ and $(c > 0)$ and $(d \geq 0)$.

Answer: 1) $ab = 0$; 2) $(ab < 0)$ and $(c = 0)$; 3) $(ab < 0)$ and $(c > 0)$ and $(d \geq 0)$.