Answer on Question #63622 – Math – Linear Algebra

Question

Given three vectors x1= transpose [2 4 8], x2= transpose [1 -1 1], x3= transpose [1 1 4]. can you write x1 as linear sum of x2 and x3?

Solution

We have $x_1 = \begin{pmatrix} 2 \\ 4 \\ 8 \end{pmatrix}$, $x_2 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$, $x_3 = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$

Write x_1 as a linear combination of x_2 and x_3:

$$x_1 = \alpha x_2 + \beta x_3$$

where α and β are some numbers.

Substituting vectors' coordinates into the equality gives

$$\begin{pmatrix} 2 \\ 4 \\ 8 \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$$

Applying the rules of operations on vectors, namely multiplication of vector by a scalar and addition of vectors, we get

$$\begin{pmatrix} \alpha \\ -\alpha \\ \alpha \end{pmatrix} + \begin{pmatrix} \beta \\ \beta \\ 4\beta \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 8 \end{pmatrix}$$

$$\begin{pmatrix} \alpha + \beta \\ -\alpha + \beta \\ \alpha + 4\beta \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 8 \end{pmatrix}$$

Equating the coordinates of the vectors, we get the system of equations:

$$\begin{cases} \alpha + \beta = 2 \\ -\alpha + \beta = 4 \\ \alpha + 4\beta = 8 \end{cases}$$

If this system has a solution, then the vector x_1 can be written as a linear combination of x_2 and x_3. Subtracting, adding the first and the second equations give

$2\alpha = -2 \implies \alpha = -1;$

$2\beta = 6 \implies \beta = 3.$

Substituting $\alpha = -1$ and $\beta = 3$ into the third equation gives:

$$-1 + 4 \cdot 3 = 11 \neq 8$$

Thus, $\alpha = -1$ and $\beta = 3$ simultaneously are not solutions of the third equation.

So the system of equations has no solution.

Thus, x_1 cannot be written as a linear combination of x_2 and x_3.

Answer: x_1 cannot be written as a linear combination of x_2 and x_3.