Answer on Question #63562 – Math – Statistics and Probability

Question

For each of the following situations, find an interval that contains (approximately or exactly) 99.73 percent of all the possible sample means. In which cases must we assume that the population is normally distributed? Why?

Solution

a) $\mu = 10,\ \sigma = 2,\ n = 25.$

$$I = \left(\mu - 3\frac{\sigma}{\sqrt{n}}, \mu + 3\frac{\sigma}{\sqrt{n}}\right) = \left(10 - 3\frac{2}{\sqrt{25}}, 10 + 3\frac{2}{\sqrt{25}}\right) = (8.8, 11.2)$$

b) $\mu = 500,\ \sigma = 0.5,\ n = 100.$

$$I = \left(\mu - 3\frac{\sigma}{\sqrt{n}}, \mu + 3\frac{\sigma}{\sqrt{n}}\right) = \left(500 - 3\frac{0.5}{\sqrt{100}}, 500 + 3\frac{0.5}{\sqrt{100}}\right) =$$
$$= (499.85, 500.15)$$

c) $\mu = 3,\ \sigma = 0.1,\ n = 4.$

$$I = \left(\mu - 3\frac{\sigma}{\sqrt{n}}, \mu + 3\frac{\sigma}{\sqrt{n}}\right) = \left(3 - 3\frac{0.1}{\sqrt{4}}, 3 + 3\frac{0.1}{\sqrt{4}}\right) = (2.925,\ 3.075)$$

d) $\mu = 100,\ \sigma = 1,\ n = 1600.$

$$I = \left(\mu - 3\frac{\sigma}{\sqrt{n}}, \mu + 3\frac{\sigma}{\sqrt{n}}\right) = \left(100 - 3\frac{1}{\sqrt{1600}}, 100 + 3\frac{1}{\sqrt{1600}}\right) =$$
$$= (99.925, 100.075)$$

By the Central Limit Theorem, the sampling distribution of the mean approaches a normal distribution.