Answer on Question #63552 – Math – Statistics and Probability

Question

The masses of packages from a particular machine are normally distributed with a mean of 200g and a standard deviation of 2g. Find the probability that a randomly selected package from the machine weighs

(i) Less than 196g

(iii) Between 198.5g and 199.5g

Solution

Denote the mass of package by *x*. Then

$$P(x_1 < x < x_2) = \int_{x_1}^{x_2} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx,$$

where μ=200, $\sigma = 2$.

Making the change of variable t= $\frac{x-\mu}{\sigma}$, we get

$$P(x_1 < x < x_2) = F\left(\frac{x_2 - \mu}{\sigma}\right) - F\left(\frac{x_1 - \mu}{\sigma}\right),$$

where $F(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt = \frac{1}{2} + \Phi(x)$, $\Phi(x) = \int_{0}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$.

For case **(i)** we get $x_1 = -\infty, x_2 = 196$. Taking into account that $\Phi(-x) = -\Phi(x)$, we have

$$P(x < 196) = F((196 - 200)/2) = F(-2) = \frac{1}{2} + \Phi(-2) = \frac{1}{2} - \Phi(2) \approx 0.5 - 0.4773 = 0.0227.$$

For case **(iii)** we get $x_1 = 198.5,\ x_2 = 199.5$.

Hence

$$\begin{aligned} P(x_1 < x < x_2) &= F\left(\frac{199.5 - 200}{2}\right) - F\left(\frac{198.5 - 200}{2}\right) \\ &= \frac{1}{2} + \Phi\left(\frac{199.5 - 200}{2}\right) - \left(\frac{1}{2} + \Phi\left(\frac{198.5 - 200}{2}\right)\right) = \\ &= \Phi\left(\frac{199.5 - 200}{2}\right) - \Phi\left(\frac{198.5 - 200}{2}\right) = \Phi(-0.25) - \Phi(-0.75) \\ &= \Phi(0.75) - \Phi(0.25) \approx 0.2734 - 0.0987 = 0.1747. \end{aligned}$$

Here the function Φ can be evaluated by means of statistical tables or using the cumulative distribution function of the standard normal distribution.

Answer: (i) 0.0227; **(iii)** 0.1747.