Answer on Question #63507 – Math – Calculus

Question

1. *Find the angle between A=2i+2j−k and B=6i−3j+2k*

Solution

$cos\alpha = \dfrac{\bar{a}\cdot\bar{b}}{|\bar{a}|\cdot|\bar{b}|}$

$\vec{a}(2;2;-1)$

$\vec{b}(6;-3;2)$

$\bar{a}\cdot\bar{b} = 2\cdot 6 + 2*(-3) + (-1)*2 = 12 - 6 - 2 = 4$

$|\bar{a}| = \sqrt{2^2 + 2^2 + (-1)^2} = 3$

$|\bar{b}| = \sqrt{6^2 + (-3)^2 + 2^2}$ =7

$cos\alpha = \dfrac{4}{3*7} = \dfrac{4}{21}$

$\alpha = 79°$

Answer: 79°

Question

2. *Determine the value of a so that A=2i+aj+k and B=4i−2j−2k are perpendicular.*

Solution

$\vec{a}(2;a;1)$

$\vec{b}(4;-2;-2)$

If vectors are perpendicular, then
2·4 + (-2)a + 1·(-2) = 0
8 – 2a –2 = 0
6 = 2a
a = 3.

Answer: a = 3.

Question

3. *Determine a unit vector perpendicular to the plane of A=2i−6j−3k and B=4i+3j−k.*

Solution

$$\begin{vmatrix} x & 2 & 4 \\ y & -6 & 3 \\ z & -3 & -1 \end{vmatrix} + D = x \cdot (-6) \cdot (-1) + 2 \cdot 3 \cdot z + 4 \cdot (-3)y - (4 \cdot (-6)z + 3 \cdot (-3)x + 2 \cdot (-1)y) + D$$

$$= 6x + 6z - 12y + 24z + 9x + 2y + D = 15x - 10y + 30z + D$$

n = (15; -10; 30) is a vector perpendicular to the plane,

$|n| = \sqrt{15^2 + (-10)^2 + 30^2} = 35.$

A unit vector perpendicular to the plane is

$n_1 = \frac{n}{|n|} = \left(\frac{15}{35}, -\frac{10}{35}, \frac{30}{35}\right) = \left(\frac{3}{7}, -\frac{2}{7}, \frac{6}{7}\right)$ or $n_2 = -\frac{n}{|n|} = -\left(\frac{15}{35}, -\frac{10}{35}, \frac{30}{35}\right) = \left(-\frac{3}{7}, \frac{2}{7}, -\frac{6}{7}\right),$

hence $n_1 = \frac{3}{7}i - \frac{2}{7}j + \frac{6}{7}k$ or $n_2 = -\frac{3}{7}i + \frac{2}{7}j - \frac{6}{7}k.$

Answer: $\frac{3}{7}i - \frac{2}{7}j + \frac{6}{7}k$ or $-\frac{3}{7}i + \frac{2}{7}j - \frac{6}{7}k.$

Question

4. *Find the work done in moving an object along a vector r=3i+2j−5k*

Solution

$\bar{F} = F_1 i + F_2 j + F_3 k$

Then work done in moving an object along a vector r=3i+2j−5k is

$A = (\bar{F}, \bar{r}) = 3F_1 + 2F_2 - 5F_3$

Answer: $3F_1 + 2F_2 - 5F_3$.

Question

5. *Given that A=2i−j+3k and B=3i+2j−k, find A·B*

Solution

If

$\vec{a}(2; -1; 3)$,

$\vec{b}(3; 2; -1)$,

then

$\vec{a} \cdot \vec{b} = 2 \cdot 3 + (-1) \cdot 2 + 3 \cdot (-1) = 6 - 2 - 3 = 1.$

Answer: 1.

Question

6. *If A=2i−3j−k and B=i+4j−2k, find (A+B)×(A−B)*

Solution

(A+B) = (2+1; (-3)+4; (-1)+(-2)) = (3; 1; -3)
(A−B) = (2-1; (-3)-4; (-1)-(-2)) = (1;-7;1)

(A+B)×(A−B) = $\begin{vmatrix} i & j & k \\ 3 & 1 & -3 \\ 1 & -7 & 1 \end{vmatrix} = \begin{vmatrix} 1 & -3 \\ -7 & 1 \end{vmatrix} i - \begin{vmatrix} 3 & -3 \\ 1 & 1 \end{vmatrix} j + \begin{vmatrix} 3 & 1 \\ 1 & -7 \end{vmatrix} k =$

= $(1 \cdot 1 - (-7) \cdot (-3)) - (3 \cdot 1 - 1 \cdot (-3))j + (3 \cdot (-7) - 1 \cdot 1)k = -20i - 6j - 22k$.

Answer: $-20i - 6j - 22k$.

Question

7. *If A=3i−j+2k, B=2i+j−k and C=i−2j+2k, find (A×B)×C*

Solution

A*B = $\begin{vmatrix} i & j & k \\ 3 & -1 & 2 \\ 2 & 1 & -1 \end{vmatrix} = \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} i - \begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} j + \begin{vmatrix} 3 & -1 \\ 2 & 1 \end{vmatrix} k = ((-1) \cdot (-1) - 1 \cdot 2)\mathrm{i} - -\mathrm{j}(3 \cdot (-1) - 2 \cdot 2) + \mathrm{k}(3 \cdot 1 - 2 \cdot (-1)) = -\mathrm{i} + 7\mathrm{j} + 5\mathrm{k}$.

(A×B) × C = $\begin{vmatrix} i & j & k \\ -1 & 7 & 5 \\ 1 & -2 & 2 \end{vmatrix} = \begin{vmatrix} 7 & 5 \\ -2 & 2 \end{vmatrix} i - \begin{vmatrix} -1 & 5 \\ 1 & 2 \end{vmatrix} j + \begin{vmatrix} -1 & 7 \\ 1 & -2 \end{vmatrix} k =$

$= (7 \cdot 2 - (-2) \cdot 5)\mathrm{i} - -((-1) \cdot 2 - 1 \cdot 5)\mathrm{j} + ((-1) \cdot (-2) - 1 \cdot 7)\mathrm{k} = 24\mathrm{i} + 7\mathrm{j} - 5\mathrm{k}$.

Answer: $24\mathrm{i} + 7\mathrm{j} - 5\mathrm{k}$

Question

8. *Determine a unit vector perpendicular to the plane of A=2i−6j−3k and B=4i+3j−k*

Solution

$$\begin{vmatrix} x & 2 & 4 \\ y & -6 & 3 \\ z & -3 & -1 \end{vmatrix} + D = 6x + 6z - 12y + 24z + 9x + 2y + D = 15x - 10y + 30z + D$$

n = (15;-10;30) is a vector perpendicular to the plane,

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A unit vector perpendicular to the plane is

$n_1 = \frac{n}{|n|} = \left(\frac{15}{35}, -\frac{10}{35}, \frac{30}{35}\right) = \left(\frac{3}{7}, -\frac{2}{7}, \frac{6}{7}\right)$ or $n_2 = -\frac{n}{|n|} = -\left(\frac{15}{35}, -\frac{10}{35}, \frac{30}{35}\right) = \left(-\frac{3}{7}, \frac{2}{7}, -\frac{6}{7}\right)$

hence $n_1 = \frac{3}{7}i - \frac{2}{7}j + \frac{6}{7}k$ or $n_2 = -\frac{3}{7}i + \frac{2}{7}j - \frac{6}{7}k$

Answer: $\frac{3}{7}i - \frac{2}{7}j + \frac{6}{7}k$ or $-\frac{3}{7}i + \frac{2}{7}j - \frac{6}{7}k$

Question

9. *Evaluate (2i−3j)·[(i+j−k)×(3i−k)]*

Solution

(2i−3j)·[(i+j−k)×(3i−k)]

(i+j−k)×(3i−k) = $\begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{vmatrix}$ = -i-2j-3k

(2i−3j)·[(i+j−k)×(3i−k)]= $(2i - 3j)\cdot(-i - 2j - 3k) = 2\cdot(-1) + (-3)\cdot(-2) + 0\cdot(-3) = -2 + 6 + 0 = 4.$

Answer: 4

Question

10. *If A=i−2j−3k, B=2i+j−k and C=i+3j−2k, evaluate (A×B)·C*

Solution

A×B = $\begin{vmatrix} i & j & k \\ 1 & -2 & -3 \\ 2 & 1 & -1 \end{vmatrix} = \begin{vmatrix} -2 & -3 \\ 1 & -1 \end{vmatrix} i - \begin{vmatrix} 1 & -3 \\ 2 & -1 \end{vmatrix} j + \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} k =$

$$= ((-2)\cdot(-1) - 1\cdot(-3))\mathrm{i} - (1\cdot(-1) - 2\cdot(-3))\mathrm{j} + (1\cdot 1 - 2\cdot(-2))\mathrm{k}$$
$$= 5\mathrm{i} - 5\mathrm{j} + 5\mathrm{k}$$

$(\mathrm{A}\times\mathrm{B})\cdot\mathrm{C} = (5\mathrm{i} - 5\mathrm{j} + 5\mathrm{k})\cdot(\mathrm{i} + 3\mathrm{j} - 2\mathrm{k}) = 5\cdot 1 + (-5)\cdot 3 + 5\cdot(-2) = 5 - 15 - 10 =$
$= -20.$

Answer: -20.