Answer on Question #63506 – Math – Calculus

Question

7. If A=5t2+tj−t3k and B=sinti−costj. Evaluate ddt(A·A)

Solution

$$\frac{d}{dt}(\vec{A}\cdot\vec{A}) = \frac{d\vec{A}}{dt}\cdot\vec{A} + \vec{A}\cdot\frac{d\vec{A}}{dt} = 2\vec{A}\cdot\frac{d\vec{A}}{dt}$$

$$\frac{d}{dt}(\vec{A}\cdot\vec{A}) = 2(5t^2\hat{\imath} + t\hat{\jmath} - t^3\hat{k}) \cdot (10t\hat{\imath} + \hat{\jmath} - 3t^2\hat{k}) = 2(50t^3 + t + 3t^5) = 6t^5 + 100t^3 + 2t.$$

Answer: $\frac{d}{dt}(\vec{A}\cdot\vec{A}) = 6t^5 + 100t^3 + 2t.$

Question

8. If A=sinui+cosuj+uk, B=cosui−sinuj−3k and C=2i+3j−k, evaluate ddu(A×(B×C)) at u=0

Solution

$$(\vec{B}\times\vec{C}) = (\mathrm{cosu}\hat{\imath} - \mathrm{sinu}\hat{\jmath} - 3\hat{k})\times(2\hat{\imath} + 3\hat{\jmath} - \hat{k}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \mathrm{cosu} & -\mathrm{sinu} & -3 \\ 2 & 3 & -1 \end{vmatrix} =$$

$$= (9 + \sin u)\hat{\imath} + (-6 + \cos u)\hat{\jmath} + (3\cos u + 2\sin u)\hat{k}$$

$$\vec{A}\times(\vec{B}\times\vec{C}) = (\mathrm{sinu}\hat{\imath} + \mathrm{cosu}\hat{\jmath} + u\hat{k})\times\left((9 + \sin u)\hat{\imath} + (-6 + \cos u)\hat{\jmath} + (3\cos u + 2\sin u)\hat{k}\right) =$$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \mathrm{sinu} & \mathrm{cosu} & u \\ 9 + \mathrm{sin}u & -6 + \mathrm{cos}u & 3\mathrm{cos}u + 2\mathrm{sin}u \end{vmatrix} =$$

$$= (6u - u\cos u + 3\cos^2 u + 2\cos u \sin u)\hat{\imath} + (9u + u\sin u - 2\sin^2 u - 3\cos u \sin u)\hat{\jmath} + (-9\cos u - 6\sin u)\hat{k}$$

$$\frac{d}{du}\vec{A}\times(\vec{B}\times\vec{C}) = (6 - \cos u + u\sin u + 2\cos 2u - 3\sin 2u)\hat{\imath} + (9 + \sin u + u\cos u - 3\cos 2u - 2\sin 2u)\hat{\jmath} + (9\sin u - 6\cos u)\hat{k}$$

$$\left(\frac{d}{du}\vec{A}\times(\vec{B}\times\vec{C})\right)_{u=0} = (6 - 1 + 0 + 2 - 0)\hat{\imath} + (9 + 0 + 0 - 3 - 0)\hat{\jmath} + (0 - 6)\hat{k} = 7\hat{\imath} + 6\hat{\jmath} - 6\hat{k}$$

Answer: $\left(\frac{d}{du}\vec{A}\times(\vec{B}\times\vec{C})\right)_{u=0} = 7\hat{\imath} + 6\hat{\jmath} - 6\hat{k}.$

Question

9. Let A=x2yzi−2xz3j−xz2 and B=4zi+yj+4x2k, find ∂2∂x∂y(A×B) at (1,0,-2)

Solution

$$(\vec{A}\times\vec{B}) = \left(x^2yz\hat{\imath} - 2xz^3\hat{\jmath} - xz^2\hat{k}\right)\times\left(4z\hat{\imath} + y\hat{\jmath} + 4x^2\hat{k}\right) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x^2yz & -2xz^3 & -xz^2 \\ 4z & y & 4x^2 \end{vmatrix} =$$

$$= (xyz^2 - 8x^3z^3)\hat{\imath} + (-4x^4yz - 4xz^3)\hat{\jmath} + (x^2y^2z + 8xz^4)\hat{k}$$

$$\frac{\partial}{\partial y}(\vec{A}\times\vec{B}) = \frac{\partial}{\partial y}(xyz^2 - 8x^3z^3)\hat{\imath} + \frac{\partial}{\partial y}(-4x^4yz - 4xz^3)\hat{\jmath} + \frac{\partial}{\partial y}(x^2y^2z + 8xz^4)\hat{k} =$$

$$= (xz^2)\hat{\imath} + (-4x^4z)\hat{\jmath} + (2x^2yz)\hat{k}$$

$$\frac{\partial^2}{\partial x\partial y}(\vec{A}\times\vec{B}) = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}(\vec{A}\times\vec{B})\right) = \frac{\partial}{\partial x}(xz^2)\hat{\imath} + \frac{\partial}{\partial x}(-4x^4z)\hat{\jmath} + \frac{\partial}{\partial x}(2x^2yz)\hat{k} = z^2\hat{\imath} - 16x^3z\hat{\jmath} + 4xyz\hat{k}$$

$$\left(\frac{\partial^2}{\partial x\partial y}(\vec{A}\times\vec{B})\right)_{(1,0,-2)} = (-2)^2\hat{\imath} - 16(1)^3(-2)\hat{\jmath} + 4\cdot 1\cdot 0\cdot(-2)\hat{k} = 4\hat{\imath} + 32\hat{\jmath} + 0\hat{k} = 4\hat{\imath} + 32\hat{\jmath}.$$

Answer: $\left(\frac{\partial^2}{\partial x\partial y}(\vec{A}\times\vec{B})\right)_{(1,0,-2)} = 4\hat{\imath} + 32\hat{\jmath}.$

Question

10. Solve d2Adt2−4dAdt−5A=0

Solution

$$\frac{d^2A}{dt^2} - 4\frac{dA}{dt} - 5A = 0$$

$$A = e^{\lambda t}$$

$$\lambda^2 - 4\lambda - 5 = 0 \to \lambda = -1 \text{ and } \lambda = 5.$$

$$A = C_1e^{\lambda_1 t} + C_2e^{\lambda_2 t} = C_1e^{-t} + C_2e^{5t},$$

where C_1 and C_2 are arbitrary real constants.

Answer: $A = C_1e^{-t} + C_2e^{5t}.$