Answer on Question #63505 – Math – Calculus

Question

1. Given that $A = \sin t\, i + \cos t\, j + tk$,

evaluate

$$\left|\frac{d^2A}{dt^2}\right|$$

Solution

If $A = \sin t\, i + \cos t\, j + tk$, then

$$\frac{dA}{dt} = (\sin t)'i + (\cos t)'j + (t)'k = \cos t\, i - \sin t\, j + k$$

$$\frac{d^2A}{dt^2} = (\cos t)'i + (-\sin t)'j + (1)'k = -\sin t\, i - \cos t\, j$$

$$\left|\frac{d^2A}{dt^2}\right| = \sqrt{(-\sin t)^2 + (-\cos t)^2} = 1.$$

Answer: $\left|\frac{d^2A}{dt^2}\right| = 1.$

Question

2. A particle moves along a curve whose parameter equations are

$$x = e^{-t}$$

$$y = 2\cos 3t$$

$$z = 2\sin 3t$$

Find the magnitude of the acceleration at $t = 0$

Solution

If $x = e^{-t}, y = 2\cos 3t, z = 2\sin 3t$, then

$$\frac{dx}{dt} = \frac{d(e^{-t})}{dt} = -e^{-t}\,;\ \frac{d^2x}{dt^2} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d}{dt}(-e^{-t}) = e^{-t}$$

$$\frac{dy}{dt} = \frac{d}{dt}(2\cos 3t) = -6\sin 3t\,;\ \frac{d^2y}{dt^2} = \frac{d}{dt}\left(\frac{dy}{dt}\right) = \frac{d}{dt}(-6\sin 3t) = -18\cos 3t$$

$$\frac{dz}{dt}=\frac{d}{dt}(2\sin 3t)=6\cos 3t\ ;\ \frac{d^2z}{dt^2}=\frac{d}{dt}\left(\frac{dz}{dt}\right)=\frac{d}{dt}(6\cos 3t)=-18\sin 3t$$

The magnitude of the acceleration:

$$|a|=\sqrt{\left(\frac{d^2x}{dt^2}\right)^2+\left(\frac{d^2y}{dt^2}\right)^2+\left(\frac{d^2z}{dt^2}\right)^2}=\sqrt{(e^{-t})^2+(-18\cos 3t)^2+(-18\sin 3t)^2}=$$

$=\sqrt{e^{-2t}+18^2}$.

The magnitude of the acceleration at $t=0$:

$|a|_{t=0}=\sqrt{e^{-2\cdot 0}+18^2}=\sqrt{1+324}=\sqrt{325}=5\sqrt{13}$.

Answer: $5\sqrt{13}$.

Question

3. A particle moves along the curve

$$x=2t^2\ ; y=t^2-4t\ ; z=3t-5$$

where t is the time. Find the components of the velocity at $t=1$ in the direction

$$\bar{l}=i-3j+2k$$

Solution

Velocity:

$$v(t)=x'(t)i+y'(t)j+z'(t)k$$

$$v(t)=(2t^2)'i+(t^2-4t)'j+(3t-5)'k=4ti+(2t-4)j+3k$$

$$v(1)=4\cdot 1i+(2\cdot 1-4)j+3k=4i-2j+3k$$

The magnitude of $\bar{l}=i-3j+2k$ is

$$|\bar{l}|=\sqrt{1^2+(-3)^2+2^2}=\sqrt{14}.$$

The dot products

$v(1)\cdot\bar{l}=(4i-2j+3k)\cdot(i-3j+2k\ )=4\cdot 1+(-2)\cdot(-3)+3\cdot 2=4+6+6=16$,

$v(1)\cdot\frac{\bar{l}}{|\bar{l}|}=\frac{1}{|\bar{l}|}v(1)\cdot\bar{l}=\frac{16}{\sqrt{14}}$.

The components of the velocity at $t=1$ in the direction $\bar{l}=i-3j+2k$ are

$(v_1)_{\bar{l}}(1)=\left(\left(v(1)\cdot\frac{\bar{l}}{|\bar{l}|}\right)\frac{\bar{l}}{|\bar{l}|}\right)_1=\left(\frac{16}{\sqrt{14}}\cdot\frac{i-3j+2k}{\sqrt{14}}\right)_1=\frac{16}{14}=\frac{8}{7}$,

$(v_2)_{\bar{l}}(1)=\left(\left(v(1)\cdot\frac{\bar{l}}{|\bar{l}|}\right)\frac{\bar{l}}{|\bar{l}|}\right)_2=\left(\frac{16}{\sqrt{14}}\cdot\frac{i-3j+2k}{\sqrt{14}}\right)_2=-3\cdot\frac{16}{14}=-\frac{24}{7}$,

$$(v_3)_{\bar{l}}(1) = \left(\left(v(1) \cdot \frac{\bar{l}}{|\bar{l}|}\right)\frac{\bar{l}}{|\bar{l}|}\right)_3 = \left(\frac{16}{\sqrt{14}} \cdot \frac{i-3j+2k}{\sqrt{14}}\right)_2 = 2 \cdot \frac{16}{14} = \frac{16}{7}.$$

Answer: $\frac{8}{7}, -\frac{24}{7}, \frac{16}{7}$.

Question

4. Determine the unit tangent at the point where $t = 2$ on the curve

$$x = t^2 + 1\,; y = 4t - 3\,; z = 2t^2 - 6t$$

Solution

The unit tangent:

$$T(t) = \frac{r'(t)}{|r'(t)|}$$

$$r'(t) = (t^2 + 1)'i + (4t - 3)'j + (2t^2 - 6t)'k = 2ti + 4j + (4t - 6)k$$

$$|r'(t)| = \sqrt{4t^2 + 16 + (4t - 6)^2}$$

$$r'(2) = 2 \cdot 2i + 4j + (4 \cdot 2 - 6)k = 4i + 4j + 2k$$

$$|r'(2)| = \sqrt{4^2 + 4^2 + 2^2} = \sqrt{16 + 16 + 4} = 6$$

$$T(2) = \frac{r'(2)}{|r'(2)|} = \frac{4}{6}i + \frac{4}{6}j + \frac{2}{6}k = \frac{2}{3}i + \frac{2}{3}j + \frac{1}{3}k$$

Answer: $T(2) = \frac{2}{3}i + \frac{2}{3}j + \frac{1}{3}k$.

Question

5. If $A = 5t^2 i + tj - t^3 k$ and $B = \sin t\, i - \cos t\, j$

evaluate

$$\frac{d}{dt}(A \cdot B)$$

Solution

If $A = 5t^2 i + tj - t^3 k$ and $B = \sin t\, i - \cos t\, j$, then

$$A \cdot B = 5t^2 \sin t - t\cos t,$$

$$\frac{d}{dt}(A \cdot B) = \frac{d}{dt}(5t^2 \sin t - t\cos t) =$$
$$= 10t\sin t + 5t^2\cos t - \cos t + t\sin t = 11t\sin t + \cos t\,(5t^2 - 1).$$

Answer: $11t\sin t + \cos t\,(5t^2 - 1)$.