

**Answer on Question #63503 – Math – Analytic Geometry  
Question**

**1. Given:**

$$A = 3i - 2j + 2k$$

**Find:** magnitude of  $A$  ( that is,  $|A|$  ).

**Solution**

$$|A| = \sqrt{3^2 + (-2)^2 + 2^2} = \sqrt{9+4+4} = \sqrt{17}$$

**Answer:**  $\sqrt{17}$

**Question**

**2. Given:**

$$A_1 = 3i - 2j + k$$

$$A_2 = 2i - 4j - 3k$$

$$A_3 = -i + 2j + 2k$$

**Find:** magnitude of  $B = 2A_1 - 3A_2 - 5A_3$

**Solution**

$$\begin{aligned} B &= 2A_1 - 3A_2 - 5A_3 = 2(3i - 2j + k) - 3(2i - 4j - 3k) - 5(-i + 2j + 2k) = \\ &= 6i - 4j + 2k - 6i + 12j + 9k + 5i - 10j - 10k = 5i - 2j + k \end{aligned}$$

$$|B| = \sqrt{5^2 + (-2)^2 + 1^2} = \sqrt{25+4+1} = \sqrt{30}$$

**Answer:**  $\sqrt{30}$

**Question**

**3. Given:**

$$A_1 = 2i - j + k$$

$$A_2 = i + 3j - 2k$$

$$A_3 = 3i + 2j + 5k$$

$$A_4 = 3i + 2j + 5k$$

$$A_4 = aA_1 + bA_2 + cA_3$$

**Find:**  $a, b, c$

### Solution

$$3i + 2j + 5k = 2ai - aj + ak + bi + 3bj - 2bk + 3ci + 2cj + 5ck$$

$$3i + 2j + 5k = (2a + b + 3c)i + (-a + 3b + 2c)j + (a - 2b + 5c)k$$

so we obtain the system of linear equations:

$$\begin{cases} 2a + b + 3c = 3 \\ -a + 3b + 2c = 2 \\ a - 2b + 5c = 5 \end{cases} \quad \begin{cases} 1 \\ 2 \\ 3 \end{cases}$$

$$\{2\} + \{3\} \Rightarrow$$

$$b + 7c = 7 \Rightarrow b = 7(1 - c)$$

$$\{2\} - 3 \cdot \{1\} \Rightarrow$$

$$-7a - 7c = -7 \Rightarrow a + c = 1 \Rightarrow a = 1 - c$$

$$\{3\} \Rightarrow$$

$$(1 - c) - 2 \cdot 7(1 - c) + 5c = 5 \Rightarrow$$

$$1 - c - 14 + 14c + 5c = 5 \Rightarrow 18c = 18 \Rightarrow$$

$$c = 1 \Rightarrow a = 1 - c = 1 - 1 = 0 \Rightarrow b = 7(1 - c) = 7(1 - 1) = 0$$

**Answer:**  $a = 0$ ,  $b = 0$ ,  $c = 1$ .

### Question

**4) Given:**

$$a \times b = ab \sin \theta$$

### Solution

It is the formula for the magnitude of the cross product.

**Answer:**  $|a \times b| = ab \sin \theta$

### Question

**5)**

A car travels 3km due north, then 5km northeast. Determine the resultant displacement

**Given:**

$$a = 3 \quad b = 5$$

**Find:**  $d$  resultant displacement

### Solution

northeast  $\Rightarrow$  angle between  $a$  and  $b$  is  $\beta = 135^\circ$

Using the law of cosines we obtain

$$\begin{aligned} d^2 &= a^2 + b^2 - 2ab \cos \beta = 9 + 25 - 30 \cos 135^\circ = 34 + 30 \cdot \sin 45^\circ = \\ &= 34 + 15\sqrt{2} \end{aligned}$$

$$d = \sqrt{34 + 15\sqrt{2}}$$

**Answer:**  $\sqrt{34 + 15\sqrt{2}}$