

Answer on Question #63499 – Math – Linear Algebra

Question

Solve the system of linear equations using Gauss elimination method:

$$\begin{aligned}x + y + z &= 2 \\2x - 3y &= 5 \\-3x + 2z &= 1.\end{aligned}$$

Solution

Write down the augmented matrix:

$$\begin{array}{ccc|c}1 & 1 & 1 & 2 \\2 & -3 & 0 & 5 \\-3 & 0 & 2 & 1.\end{array}$$

Reduce this to row echelon form by using elementary row operations:

a) $-2 * R1 + R2 \rightarrow R2$; $3 * R1 + R3 \rightarrow R3$:

$$\begin{array}{ccc|c}1 & 1 & 1 & 2 \\0 & -5 & -2 & 1 \\0 & 3 & 5 & 7\end{array}$$

b) $3 * R2 + 5 * R3 \rightarrow R3$:

$$\begin{array}{ccc|c}1 & 1 & 1 & 2 \\0 & -5 & -2 & 1 \\0 & 0 & 19 & 38\end{array}$$

c) $1/19 * R3 \rightarrow R3$:

$$\begin{array}{ccc|c}1 & 1 & 1 & 2 \\0 & -5 & -2 & 1 \\0 & 0 & 1 & 2.\end{array}$$

Changing back to system of equations, we have:

$$\begin{aligned}x + y + z &= 2 \\-5y - 2z &= 1 \\z &= 2.\end{aligned}$$

Now, the solution can be obtained by back substitution:

$$z = 2,$$

$$y = \frac{(1 + 2z)}{-5} = (1 + 2 * 2)/(-5) = -1,$$

$$x = 2 - y - z = 2 - (-1) - 2 = 1.$$

The solution is $(x, y, z) = (1, -1, 2)$.

Answer: $(x, y, z) = (1, -1, 2)$.