

### Answer on Question #63447 – Math – Statistics and Probability

The following data relates to daily bill (in Kshs) on consumption of a certain commodity for 60 households

Daily bills (Kshs)	No. of households
10 – 20	6
20 – 30	7
30 – 40	11
40 – 50	10
50 – 60	6
60 – 70	5
70 – 80	9
80 – 90	3
90 – 100	3

- i) Calculate the mean
- ii) Calculate the median
- iii) Calculate the mode
- iv) Calculate the standard deviation
- v) Calculate the coefficient of skewness
- vi) Comment on the skewness of this distribution
- vii) Calculate the coefficient of variation

#### Solution (i)

The mean is

$$\begin{aligned}\bar{x} &= \frac{\sum_{i=1}^n x_i f_i}{N} \\ &= \frac{15 \cdot 6 + 25 \cdot 7 + 35 \cdot 11 + 45 \cdot 10 + 55 \cdot 6 + 65 \cdot 5 + 75 \cdot 9 + 85 \cdot 3 + 95 \cdot 3}{60} = 49.5\end{aligned}$$

**Answer (i):** The mean is  $\bar{x} = 49.5$

#### Solution (ii)

The median is in the class where the cumulative frequency reaches half the sum of the absolute frequencies. That is to say, the median is within the class  $\frac{N}{2}$

$$M_e = L_i + \frac{\frac{N}{2} - F_{i-1}}{f_i} a_i$$

$L_i$  is the lower limit of the median class.

$\frac{N}{2}$  is half the sum of the absolute frequency.

$F_{i-1}$  is the absolute frequency immediately below the median class.

$a_i$  is the width of the class containing the median class.

	fi	Fi
10-20	6	6
20-30	7	13
30-40	11	24
40-50	10	34
50-60	6	40
60-70	5	45
70-80	9	54
80-90	3	57
90-100	3	60
	60	

$$a_i = 10$$

$$\frac{N}{2} = \frac{60}{2} = 30$$

Median class: 40-50

$$L_i = 40$$

$$f_i = 10$$

$$F_{i-1} = 24$$

$$M_e = 40 + \frac{30 - 24}{10} \cdot 10 = 40 + 6 = 46$$

**Answer (ii):** The median is  $M_e = 46$ .

**Solution (iii)**

The mode is the most repeated value in a distribution.

Mode for Grouped Data Formula

$$M_o = L_i + \frac{f_i - f_{i-1}}{(f_i - f_{i-1}) + (f_i - f_{i+1})} \cdot a_i$$

$L_i$  is the lower limit of the modal class.

$f_i$  is the absolute frequency of the modal class.

$f_{i-1}$  is the absolute frequency immediately below the modal class.

$f_{i+1}$  is the absolute frequency immediately after the modal class.

$a_i$  is the width of the class containing the modal class.

10 – 20	6
20 – 30	7
30 – 40	11
40 – 50	10
50 – 60	6
60 – 70	5
70 – 80	9
80 – 90	3
90 – 100	3

Modal class 30-40

$$M_o = L_i + \frac{f_i - f_{i-1}}{(f_i - f_{i-1}) + (f_i - f_{i+1})} \cdot a_i = 30 + \frac{11 - 7}{(11 - 7) + (11 - 10)} \cdot 10 =$$

$$= 30 + \frac{4}{5} \cdot 10 = 38$$

**Answer(iii):** the mode is  $M_o = 38$ .

**Solution (iv)**

To calculate the standard deviation,

use the following formula:

$$\sigma = \sqrt{\frac{\sum_{i=1}^n x_i^2 f_i}{N} - \bar{x}^2}$$

$x_i$	$f_i$	$x_i^2 f_i / N$
15	6	22.5
25	7	72.91667
35	11	224.5833
45	10	337.5
55	6	302.5
65	5	352.0833
75	9	843.75
85	3	361.25
95	3	451.25
	The sum is	2968.333

$$\sigma = \sqrt{\frac{\sum_{i=1}^n x_i^2 f_i}{N} - \bar{x}^2} = \sqrt{2968.333 - (49.5)^2} = 22.76$$

**Answer:** The standard deviation is  $\sigma = 22.76$

**Solution (v)**

The coefficient of skewness we calculate using the following formula

$$K_s = \frac{\mu_3}{\sigma^3} = \frac{\sum_{i=1}^n (x_i - \bar{x})^3 f_i}{N\sigma^3}$$

$\sigma = 22.76$  and  $\bar{x} = 49.5$  we found before, find the sum

$x_i$	$x_i - \bar{x}$	$(x_i - \bar{x})f_i$	$\frac{(x_i - \bar{x})^3 f_i}{\sigma^3 N}$
15	-34.5	-246382	-0.348289604
25	-24.5	-102943	-0.145521871
35	-14.5	-33534.9	-0.047405493
45	-4.5	-911.25	-0.001288159
55	5.5	998.25	0.001411144
65	15.5	18619.38	0.026320678
75	25.5	149232.4	0.210957528
85	35.5	134216.6	0.189730998
95	45.5	282589.1	0.399472991
		The sum is	0.285388211

$$\text{So } K_s = \frac{\mu_3}{\sigma^3} = \frac{\sum_{i=1}^n (x_i - \bar{x})^3 f_i}{N\sigma^3} = 0.285.$$

**Answer (v):** the coefficient of skewness is  $K_s = 0.285$

**Solution (vi)**

*Positive skew:* The right tail is longer; the mass of the distribution is concentrated on the left of the figure. The distribution is said to be *right-skewed*, *right-tailed*, or *skewed to the right*.

**Answer (vi):** right-skewed.

**Solution (vii)**

The coefficient of variation is defined as the ratio of the standard deviation  $\sigma$  to the mean  $\bar{x}$

$$c_v = \frac{\sigma}{\bar{x}} = \frac{22.76}{49.5} = 0.46$$

**Answer (vii):** The coefficient of variation is  $c_v = 0.46$ .